Disordered Josephson junction chains: Anderson localization of normal modes and impedance fluctuations

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Thanks to: F. Hekking, W. Guichard, V. Golovach, G. Crisan

physical phenomenon
(its varoius aspects)

Anderson localization
(anisotropy of the disorder,
wave polarization,
nonlinearities, etc.)

models

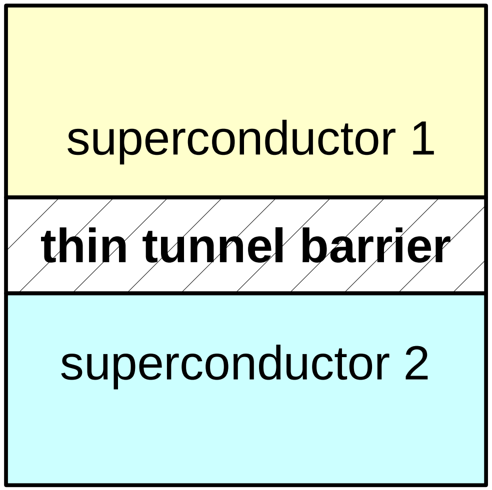
Schrödinger equation,
wave equation,
tight-binding, etc.

physical systems

Electrons, light,
cold atoms, ultrasound,
Josephson junction chains

The subject of the present talk

Josephson junction

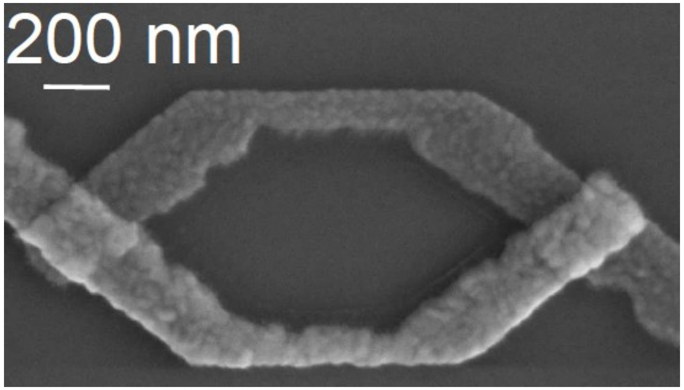


Josephson current: $I_{1\to 2} = I^c \sin(\phi_1 - \phi_2)$

critical current
(characterizes
the structure
of the contact)
nanoamperes

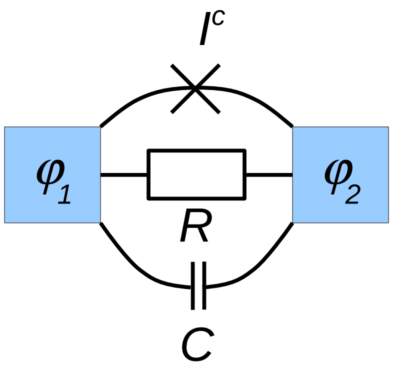
superconducting
phases
(characterize the state
of each island)

- Superconducting QUantum Interference Devices
  (the most sensitive magnetometers)
- Qbits
- metrology (frequency ↔ voltage)



SQUID (image from W. Guichard)

RCSJ model



The total current between the islands

$$I_{1\to 2} = I^c \sin(\phi_1 - \phi_2) + \frac{V_1 - V_2}{R} + C\,\frac{d(V_1 - V_2)}{dt}$$

Josephson (Cooper pairs) — normal (quasiparticles) — displacement (capacitance)

$$\frac{d\phi_n}{dt} = -2eV_n$$ gauge invariance in the superconductor

Classical regime:

$gigahertz$ inverse time of transfer of one Cooper pair

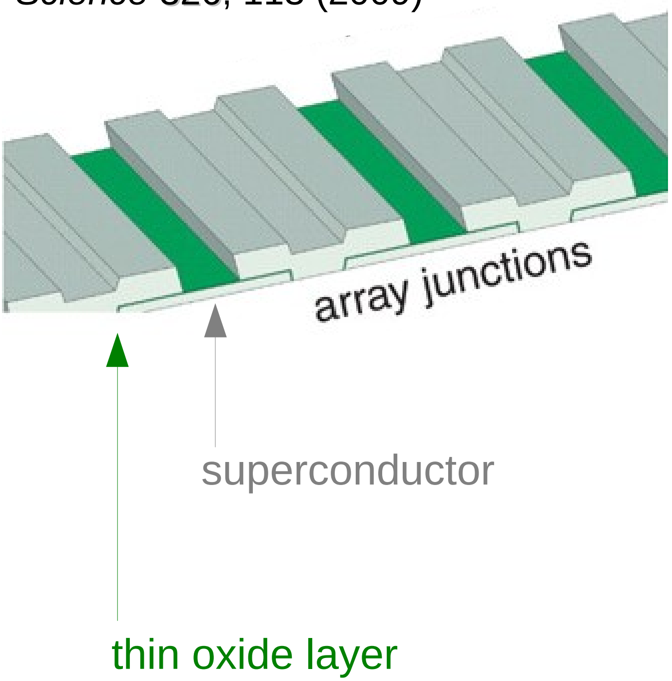
$$\frac{I^c}{2e} \gg \frac{(2e)^2}{2C}$$

charging energy due to capacitance between the islands $femtofarads$

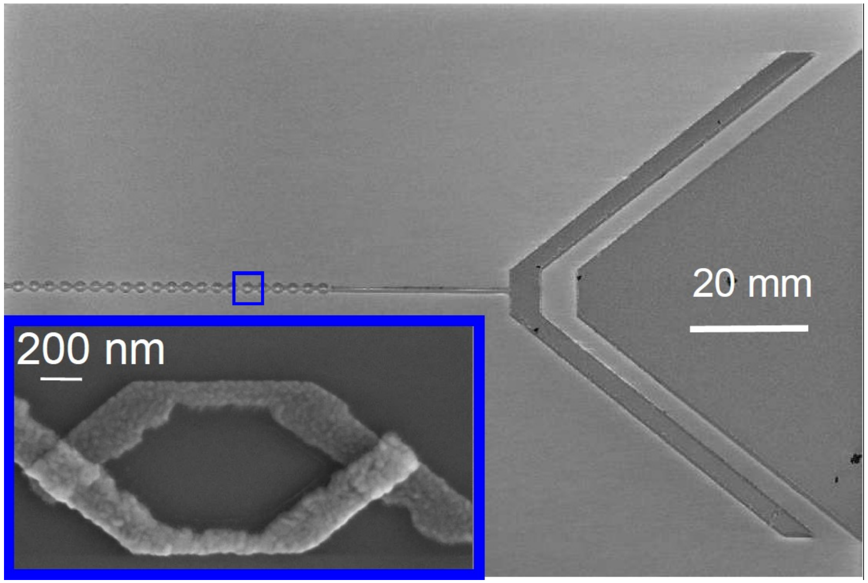
Otherwise: strong Coulomb blockade (charge discreteness matters)

Josephson junction chains

V. Manucharyan *et al.*,
Science 326, 113 (2009)

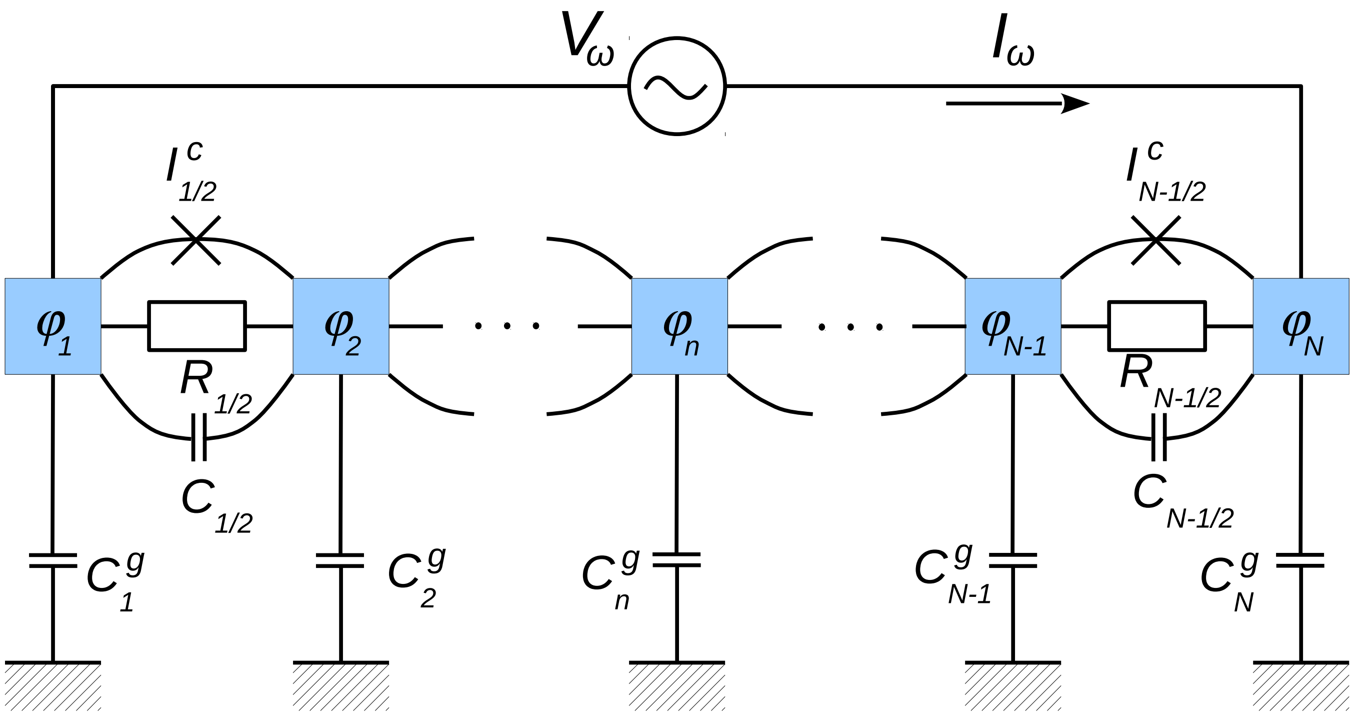


SQUID chain (image from W. Guichard):



- large impedance with little dissipation Maslyuk *et al.*, PRL **109**, 137002 (2012)
- control over quantum coherence (phase slips) Pop *et al.*, Nature Phys. **6**, 589 (2010)

Josephson junction chains



a small capacitance between the islands and the ground

$$I_{n-1\to n} - I_{n\to n+1} = C^g_n \frac{dV_n}{dt}$$

nonlinear "wave" equation

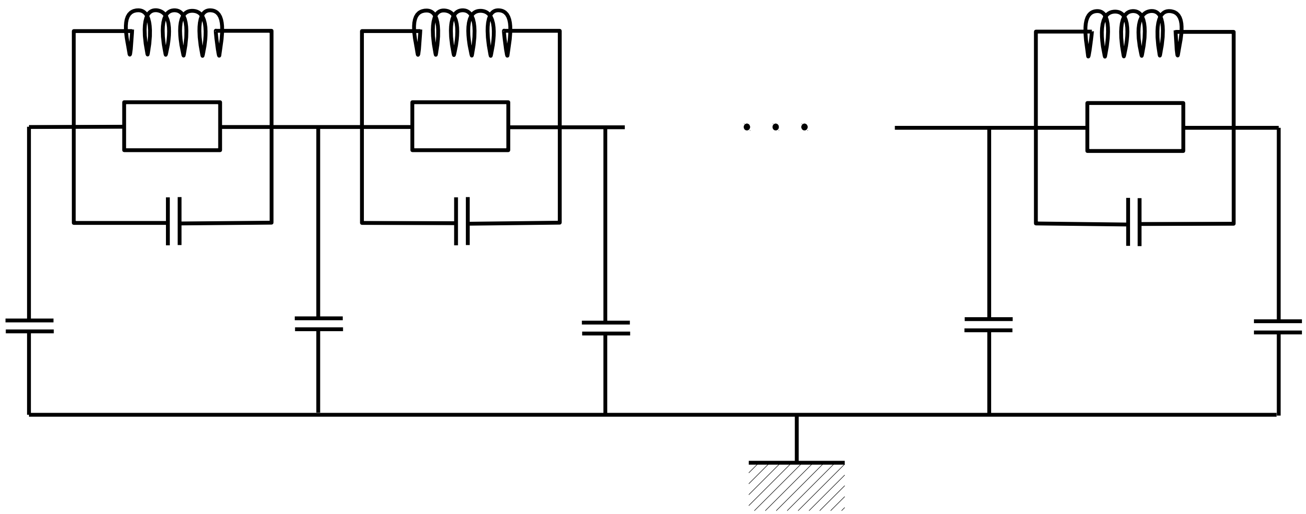
the three contributions

because of $I^c \sin(\phi_n - \phi_{n+1})$

Small oscillations of the phase

$$\sin(\phi_n - \phi_{n+1}) \to \phi_n - \phi_{n+1}$$

Josephson current $\to$ inductance



$$Y_{n-1/2}(V_n - V_{n-1}) + Y_{n+1/2}(V_n - V_{n+1}) - i\omega C_n^g V_n = 0$$ linear “wave” equation

$$Y_{n+1/2}(\omega) = -\frac{1}{i\omega L_{n+1/2}} - i\omega C_{n+1/2} + \frac{1}{R_{n+1/2}}$$ complex admittance of the junction

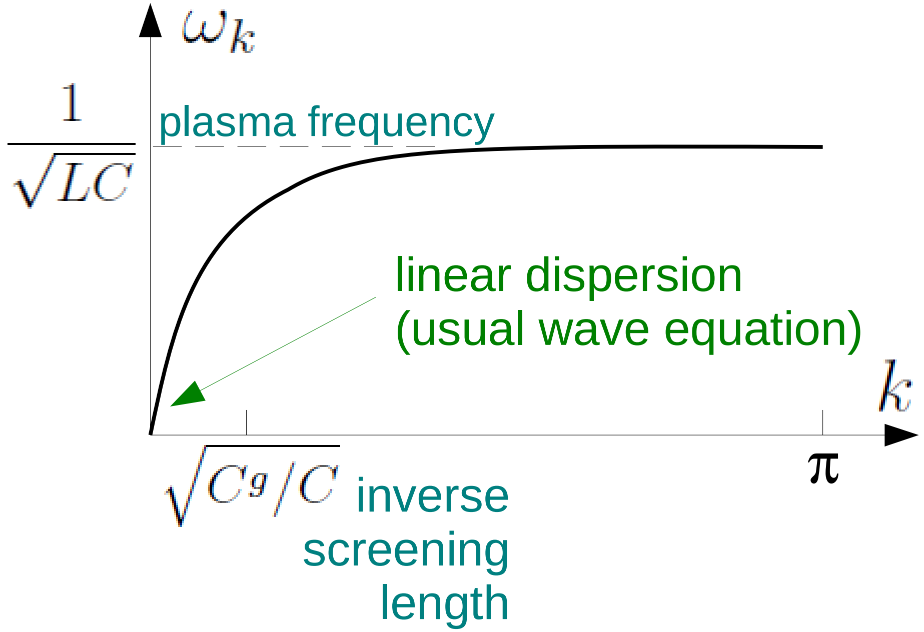
dissipation!

Non-Hermitian quadratic eigenvalue problem

No disorder, no dissipation

Infinitely long chain: $V_n \propto e^{ikn}$

$$\omega_k = \frac{1}{\sqrt{LC}}\sqrt{\frac{4\sin^2(k/2)}{4\sin^2(k/2)+C^g/C}}$$



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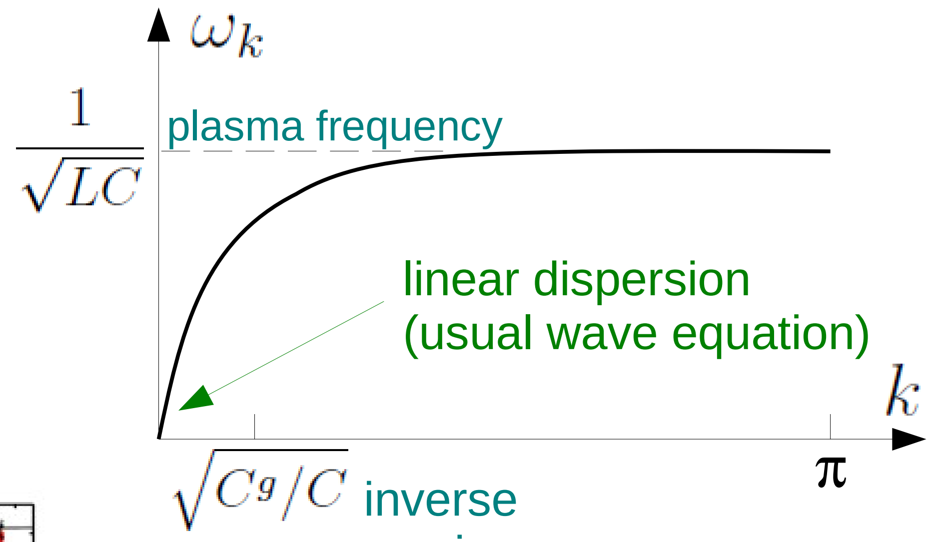
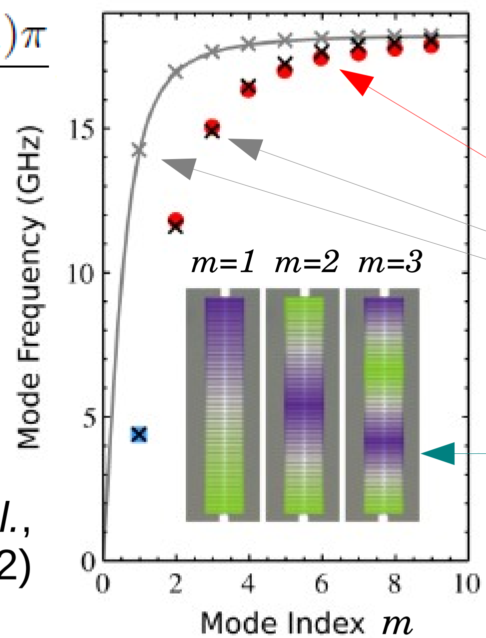


Finite length, N junctions:

$$k = 0, \frac{\pi}{N}, \frac{2\pi}{N}, \ldots, \frac{(N-1)\pi}{N}$$

80 junctions:



experimental frequencies

theoretical calculation

$C^g / C \approx 10^{-3}$

mode profiles

Masluk *et al.*,
PRL **109**, 137002 (2012)

Disorder, no dissipation

Critical current, junction capacitance ∝ **junction area** – the main source of disorder

$$L_{n+1/2} = \frac{L}{1+\zeta_n}, \quad C_{n+1/2} = C(1+\zeta_n), \quad \left\langle \zeta_n^2 \right\rangle = \sigma_S^2 \ll 1$$

weak relative fluctuations of the junction areas

$$C_n^g = C^g(1+\eta_n), \quad \left\langle \eta_n^2 \right\rangle = \sigma_g^2$$

weak relative fluctuations of the ground capacitances

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Long chains: the inverse localization length from the DMPK equation

$$\frac{1}{\xi} = \frac{\sigma_S^2 + \sigma_g^2}{2} \tan^2 \frac{k}{2}$$

at $k \to 0$ goes as k^2 (standard behavior for Goldstone modes)
at $k \to \pi$ diverges

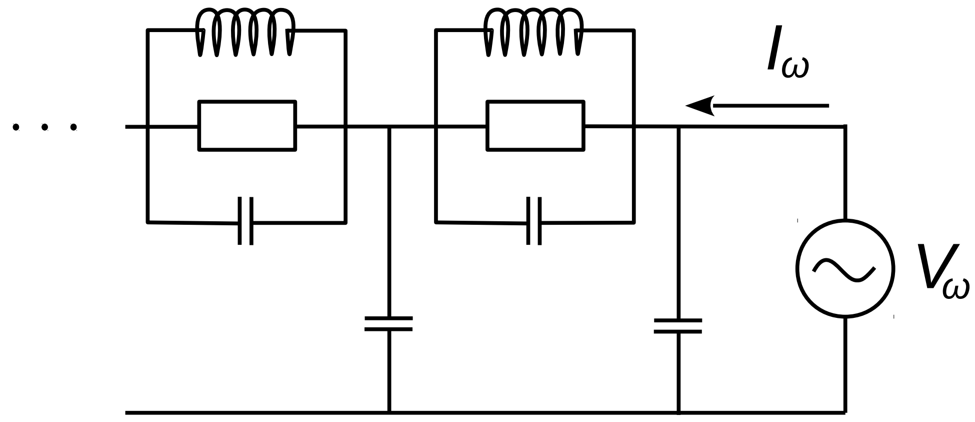
Short chains $N \ll \xi$: random perturbative shifts of the discrete frequencies

$$\left\langle \delta\omega_k^2 \right\rangle = \frac{3/8}{LC} \, \frac{\sigma_S^2 + \sigma_g^2}{N} \, \frac{(C^g/C)^2 \, 4\sin^2(k/2)}{[4\sin^2(k/2) + C^g/C]^3}$$

motional narrowing

Basko & Hekking, PRB **88**, 094507 (2013)

Disorder and dissipation



The system must be driven

external AC voltage

Impedance of the semi-infinite chain $Z(\omega) = \frac{V_\omega}{I_\omega}$

↕ *one-to-one correspondence*

Reflection coefficient of the transmission line

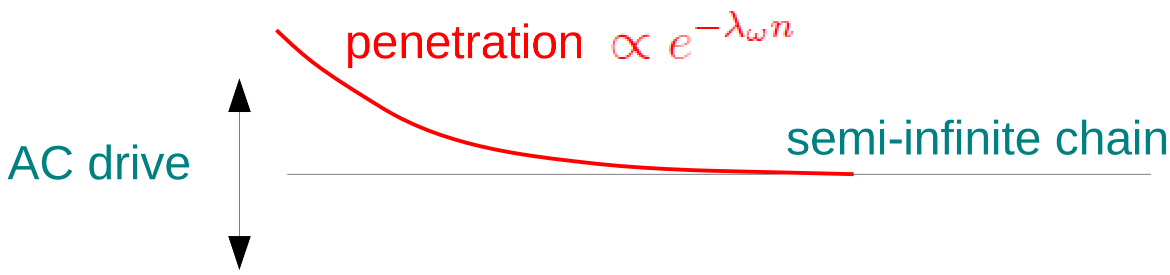
Statistics of reflection coefficient in the disordered Helmholz equation with absorption:

one mode {
Freilikher, Pustilnik & Yurkevich, *PRL* **73**, 810 (1994) — moments
Pradhan & Kumar, *PRB* **50**, 9644 (1994)

many modes {
Beenakker, Paasschens & Brouwer, *PRL* **76**, 1368 (1996)
Bruce & Chalker, *J. Phys. A* **29**, 3761 (1996)

} probability distribution

Localization and absorption



inverse penetration depth ≡ spatial Lyapunov exponent $\lambda_\omega(\sigma^2, Q^{-1})$

disorder strength (σ²), absorption strength (Q^{-1})

Taylor series (weak disorder, weak absorption)

$$\lambda_\omega(\sigma^2, Q^{-1}) = a(\omega)\,\sigma^2 + b(\omega)\,Q^{-1} + \ldots$$

$a(\omega)\,\sigma^2$: $1/\xi(\omega)$ — inverse localization length without absorption

$b(\omega)\,Q^{-1}$: $\kappa(\omega)$ — inverse absortion length without disorder

enter additively

$$Q \equiv \frac{RC}{\sqrt{LC}}$$

$$\kappa = \frac{1}{2Q}\,\frac{\omega^2 LC\sqrt{C^g/C}}{(1-\omega^2 LC)\sqrt{1-\omega^2 LC-\omega^2 LC^g/4}}$$

Both $1/\xi, \kappa \ll k$. What happens for $\kappa \gg 1/\xi$ and $\kappa \ll 1/\xi$?

Absorption by the eigenmodes

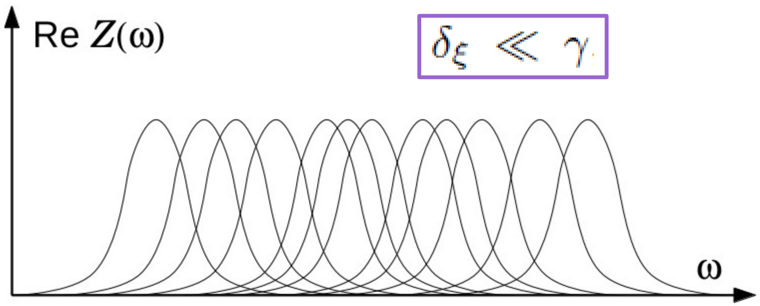
Absorption of the transmission line

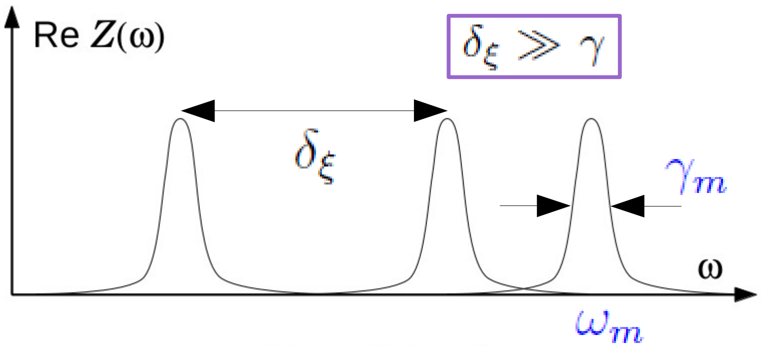
$$\mathrm{Re}\, Z(\omega) \sim \sum_m \frac{\gamma_m A_m}{(\omega-\omega_m)^2+{\gamma_m}^2}$$

strength of the mth eigenmode at the location of the drive

$\omega_m + i\gamma_m$ complex eigenvalue of the non-Hermitian problem

only modes within length ~ ξ effectively contribute





Mode spacing within a localization length: $\delta_\xi \approx \left(\frac{\xi}{2\pi}\frac{dk}{d\omega_k}\right)^{-1}$

determined by disorder only

density of states without disorder

Mode broadening: $\gamma \approx \frac{d\omega_k}{dk}\kappa$

determined by absorption only

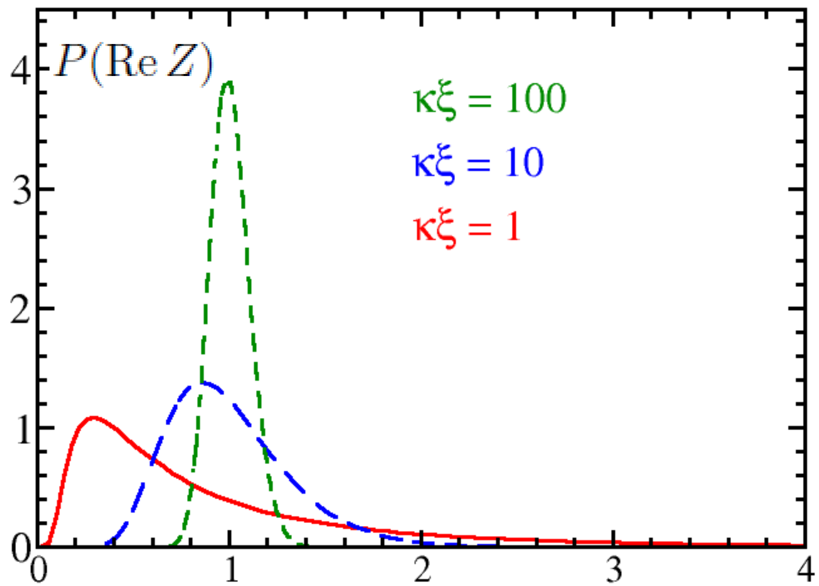
dispersion law without disorder

$$\frac{\gamma}{\delta_\xi} = \frac{\kappa\xi}{2\pi}$$

the main control parameter

Impedance statistics

Probability distribution
of the normalized resistance



normalized resistance $\dfrac{\mathrm{Re}\,Z(\omega)}{\mathrm{Re}\,Z_0(\omega)}$

impedance without disorder

Absorption dominates: $\kappa \gg 1/\xi$

$$P(Z) \sim \exp\left(-\frac{\kappa\xi}{4}\,\frac{|Z-Z_0|^2}{|Z_0|^2}\right)$$

small mesoscopic fluctuations
calculated from the DMPK equation

Localization dominates: $\kappa \ll 1/\xi$

$$P(\mathrm{Re}\,Z) \sim \frac{\sqrt{\kappa\xi\,\mathrm{Re}\,Z_0}}{(\mathrm{Re}\,Z)^{3/2}}$$

universal power-law tail
here obtained from numerics

Pradhan & Kumar, *PRB* **50**, 9644 (1994)

Conclusions

1. Small phase oscillation in JJ chains $\rightarrow$
   a wave-like system with controllable disorder and absorption

2. Relevant observable quantity: impedance $Z(\omega)$

3. Absorption dominates over localization: weak Gaussian fluctuations

4. Localization dominates over absorption: universal power-law tail
   in the distribution function