Light superdiffusion in Lévy glass

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From diffusion to superdiffusion

Gaussian distribution:
Fast decaying tails $\rightarrow$ finite variance

Cauchy distribution:
Slow decaying tails $\rightarrow$ infinite variance
A new central limit theorem

The sum of identically distributed and independent random variables with finite variance converges toward a Gaussian.

BUT

The sum of identically distributed and independent random variables converges toward a $\alpha$-stable Lévy distribution!!!

\[ P(k, t) \propto e^{-|\gamma k|^\alpha} \]

\[ 0 < \alpha \leq 2 \]

\[ P(r, t) \propto \frac{1}{r^{\alpha+1}} \]
Superdiffusion happens!

Pulsar scintillation

Banknotes spreading

Stock market fluctuations

Animal food search patterns
G.M. Viswanathan et al., Nature 381 413 1996.
Economy, geology, ethology, hydrology and astronomy are all observational sciences.

No possibility for controlled experiment.

No real possibility to test different theories and models.
How to make experiments?

2D Turbulent media


Light in hot atomic vapour


Elegant experiments, but they don't allow to control the transport parameters.

Why not to use light?
Light and random walks

Multiple scattering can be mapped on a random walk.

\[ P(\ell) \propto e^{-\ell/\ell_s} \]

\[ \langle r^2 \rangle = 2I_0 Dt \]
How to make light superdiffuse?

Lévy flights were introduced by Mandelbrot as a random walk on a fractal.

Maybe we can study light scattering from fractal-like structures?

It doesn't work: small features scatter less than big features.
Let's engineer particle density

\[ \ell = \frac{1}{\sigma n} \]

We still have one degree of freedom: the local density of scattering particles.
Building a Lévy glass

TiO2 nanoparticles (scatterers) + Sodium silicate (matrix) + Glass spheres (spacers)

Approximatively a fractal support for the scatterers.

\[ P(d) \rightarrow P(\ell) \]
The void size distribution

We impose: \( P(d) \propto \frac{1}{d^{\beta+1}} \) and we want to know \( \alpha \)

\[
P(b) = \int_0^\infty P(b|d) P(d) \, dd \propto b^{1-\beta}
\]

i.e. \( \beta = \alpha + 1 \) \quad We have our recipe!
Does it really work?

Let's create a sample with $\beta = 2$ (i.e. $\alpha = 1$)

$$T = \frac{1}{1 + AL^{\alpha/2}}$$

- $\alpha = 0.948$ (Lévy transport)
- $\alpha = 2$ (Diffusive transport)
Transmission profiles

Diffusive

Lévy
Profile vs. $\alpha$

Experiment

Theory
Conclusions

- There is the need for controlled experiments on superdiffusion.
- Light is an optimal tool to study superdiffusive processes.
- We can create systems where light superdiffuse.
- Controlling the diameters distribution of voids we can tune $\alpha$. 
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(questions are extremely welcome!)

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Fractional derivatives

\[ k^2 \tilde{\rho}(k, t) \leftrightarrow \frac{\partial^2 \rho(r, t)}{\partial r^2} \]

\[ k^\alpha \tilde{\rho}(k, t) \leftrightarrow \frac{\partial^\alpha \rho(r, t)}{\partial r^\alpha} \]

\[ \dot{\rho}(r, t) = D_\alpha \nabla^\alpha \rho(r, t) \]

Lévy flights lead to superdiffusion

\[ \left \langle r^2 \right \rangle = D_\alpha t^{\frac{2}{\alpha}} \quad 0 < \alpha \leq 2 \]
Lévy flights vs. Lévy walks

Lévy flights: each step in a unit time → arbitrary high speed are possible → unphysical

\[ \langle r^2 \rangle = D_\alpha t^{\frac{2}{\alpha}} \quad 0 < \alpha \leq 2 \]

Lévy walks: each step is performed at constant speed

\[ \langle r^2 \rangle = D_\alpha t^{3-\alpha} \quad 1 \leq \alpha \leq 2 \]
Natural Lévy glasses?

Volcanic stones

Beer bubbles
(P. J. Plath, Discrete Dynamics in Nature and Society, 2006)