Anderson Localization of BECs in the Disordered Bose-Hubbard Model

Mott Lobes, Superfluidity and Bose Glass

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Outline

Introduction:
Anderson vs. Mott localization
Bose glass in the strongly vs. weakly interacting region

Disordered Bose-Hubbard model:
Stochastic mean field theory for the average, local OP:
Mott insulator vs. finite compressibility with/without BEC

Transport theory of superfluidity and the Bose glass transition

Conclusion
**Intro: Anderson localization in ultracold atomic gases**


“Localization [..], very few believed it at the time, and even fewer saw its importance, among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it.”

P.W. Anderson, Nobel Lecture, 1977

**Scaling theory (1979): „Gang of Four“ AALR**

Wave interference effect, enhanced backscattering

Occurs in any random wave system.

Iterated backscattering →

**Selfconsistent theory**

Vollhardt, Wölfle (1980) → Strong localization

**AL in 1D ultracold bosonic gases (non-interacting):**

J. Billy, ... A. Aspect, Nature **453**, 891 (2008)

Intro: Mott insulator in ultracold gases (bosonic)

N. Mott: Hopping on a lattice forbidden by repulsive on-site energy barrier

Sir Nevill Francis Mott
Nobel Prize 1977

Mott insulator: $2d \frac{t}{U} > 1$
integer occupation number $n$

**Intro: Bose glass with weak vs strong interactions**

- **Strong interactions**
  - Mott regime
  - \( \langle N \rangle = 3 \)
  - \( \langle N \rangle = 2 \)
  - \( \langle N \rangle = 1 \)

- **Weak interactions**
  - Bogoliubov regime
  - Anderson localization of weakly interacting BEC wave function in a random potential.

- **Mott phase**: \( \langle \psi \rangle = 0 \), \( \kappa = 0 \) (incompressible)
- "Bose glass": \( \langle \psi \rangle = 0 \), \( \kappa > 0 \) no info abt transport
- Disorder-induced superfluidity ???
- **Theorem of inclusions**: no direct transition Mott \( \leftrightarrow \) superfluid

- **Gross-Pitaevskii**:
  - non-linear Schrödinger eq. + disorder
- **Many-body localization**
- **Competition**: interference \( \leftrightarrow \) dephasing

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- Fisher et al., PRB 40, 546 (1989)
- Bisbort, Hofstetter, EPL 86, 50007 (2009)
- Polet, Prokof'ev, Troyer, PRL 103, 140402 (2009)

**Theory**: Basko, Aleiner, Al'tshuler; Flach, …

**Experiment**: Aspect, Inguscio
Intro: Bose glass with weak vs strong interactions

- **Mott phase**: \( \langle \psi \rangle = 0 \), \( \kappa = 0 \) (compressible)
- „Bose glass“: \( \langle \psi \rangle = 0 \), \( \kappa > 0 \)  no info abt transport
- Disorder-induced superfluidity ???
- **Theorem of inclusions**: no direct transition Mott \( \leftrightarrow \) superfluid

Weak interactions
Bogoliubov regime

Anderson localization of weakly interacting BEC wave function in a random potential.

Experiment:
- Gross-Pitaevskii: non-linear Schrödinger eq.
- Many-body localization
- Competition: interference dephasing
  - Theory: Basko, Aleiner, Al’tshuler; Flach,...
  - Experiment: Aspect, Inguscio...

Fisher et al., PRB 40, 546 (1989)
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\[
\mu / U \quad \text{Strong interactions} \quad \text{Weak interactions}
\]

\[
\begin{align*}
\langle \psi \rangle = 0 & \quad <N> = 3 \\
\langle \psi \rangle = 0 & \quad <N> = 2 \\
\langle \psi \rangle = 0 & \quad <N> = 1
\end{align*}
\]
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Disordered Bose-Hubbard: stochastic mean field

\[ H = \sum_i \left( \varepsilon_i - \mu + \frac{U}{2} (b_i^\dagger b_i - 1) \right) b_i^\dagger b_i - t \sum_{\langle ij \rangle} b_i^\dagger b_j \]

\[ H = \sum_i \left[ \left( \varepsilon_i - \mu + \frac{U}{2} (b_i^\dagger b_i - 1) \right) b_i^\dagger b_i - t \sum_{j.n.n.i} \left( \psi_j^* b_i + \psi_j b_i^\dagger - \psi_j^* \psi_i \right) \right] - t \sum_{\langle ij \rangle} \delta_i \delta_j \]

\[ b_i = \langle 0 | b_i | 0 \rangle + \delta_i \]

\[ \psi_i = \langle 0 | b_i | 0 \rangle \]
Disordered Bose-Hubbard: stochastic mean field

\[ H = \sum_i \left( \varepsilon_i - \mu + \frac{U}{2} (b_i^\dagger b_i - 1) \right) b_i^\dagger b_i - t \sum_{\langle ij \rangle} b_i^\dagger b_j \]  

\[ H = \sum_i \left[ \left( \varepsilon_i - \mu + \frac{U}{2} (b_i^\dagger b_i - 1) \right) b_i^\dagger b_i - t \sum_{j.n.n.i} (\psi_j^* b_i + \psi_j b_i^\dagger - \psi_j^* \psi_i) \right] - t \sum_{\langle ij \rangle} \delta_i^\dagger \delta_j \]

\[ b_i = \langle 0|b_i|0 \rangle + \delta_i \]

\[ \psi_i = \langle 0|b_i|0 \rangle \]

Diagonalizing local Hamiltonian exactly in Fock space:

\[ \psi = \psi(\varepsilon, \mu, t/U) \]

\[ E_{i\alpha} \quad \text{many-body eigenenergies} \]
Disordered Bose-Hubbard: stochastic mean field

\[ H = \sum_i \left( \varepsilon_i - \mu + \frac{U}{2} (b_i^\dagger b_i - 1) \right) b_i^\dagger b_i - t \sum_{\langle ij \rangle} b_i^\dagger b_j \]

Adding disorder: random \( \varepsilon_i \)

Local Fock space

\[ b_i = \langle 0 | b_i | 0 \rangle + \delta_i \]

\[ \psi_i = \langle 0 | b_i | 0 \rangle \]

Diagonalizing local Hamiltonian exactly in Fock space:

\[ \psi_i = \psi(\varepsilon_i, \mu, t / U, \psi_j) \]

\[ E_{i\alpha} \] many-body eigenenergies
Disordered Bose-Hubbard: stochastic mean field

\[ \mathcal{P}_\psi(\{\psi_i \}) = \prod_{i}^{N} \int d\varepsilon_i \ P_\varepsilon(\varepsilon_i) \ \delta(\psi_i - \psi(\{\varepsilon_i\}, \mu, t, U)) \]

\[ \mathcal{P}_\psi(\{\psi_i \}) = \prod_{i=1}^{N} P_\psi(\psi_i) \quad \text{local approximation for } \psi \text{-distribution} \]

\[ P_\psi(\psi_i) = \int d\varepsilon \ P_\varepsilon(\varepsilon) \ \prod_{j, \text{nn.i}}^{z} \int d\psi_j \ P_\psi(\psi_j) \ \delta(\psi_i - \psi(\varepsilon, \mu, t, U, \{\psi_j \})) \]
Disordered Bose-Hubbard: stochastic mean field

Disorder-induced superfluid OP $\rightarrow$ superfluidity ???

re-entrant, disorder-induced, superfluid OP $\rightarrow$ superfluidity ???

$W/U=0.6$
Disordered Bose-Hubbard: stochastic mean field

\[ \langle \psi \rangle \approx 0 \quad \kappa > 0 \]

but true phase transition
\[ \langle \psi \rangle = 0 \leftrightarrow \langle \psi \rangle > 0 \]

\textbf{not} possible!

re-entrant, disorder-induced, superfluid OP
\[ \rightarrow \text{superfluidity ???} \]
Disordered Bose-Hubbard: stochastic mean field

Local OP distribution functions

$P_{\psi}(\psi_i)$

$\langle \psi_i \rangle$
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Local many-body bands:

\[ \Delta n_\alpha = n_\alpha - n_0 \]

\[ \Delta n_\alpha = 1 \]

\[ n_0 = 1 \]
\[ n_0 = 2 \]

\[ t=0 \]

no disorder: \[ W/U = 0 \]

\[ t/U = 0.021 \]

The spectra result from a combination of

- local charging energy:
- discreteness of particle number \( n_\alpha \)

\[ E_\alpha = (\varepsilon_i - \mu)n_\alpha + \frac{U}{2} (n_\alpha - 1)n_\alpha \]
Including non-local fluctuations

\[ H = \sum_i \left[ (\varepsilon_i - \mu + \frac{U}{2}(b_i^\dagger b_i - 1))b_i^\dagger b_i - t \sum_{j,nn,i} (\psi_j^* b_i + \psi_j b_i^\dagger - \psi_j^* \psi_i) \right] - t \sum_{\langle ij \rangle} (b_i^\dagger - \psi_i^+) \delta_i^+ \delta_j^- \]

fluctuation operator: \[ \delta_i^+ = b_i^+ - \psi_i^+ \]
\[ \rightarrow \text{no dynamics in the local MF ground states} \mid i0 \rangle \]

Representation in product basis of exact local many-body states:

\[ \{ \mid 1\alpha_1 \rangle \mid 2\alpha_2 \rangle \ldots \mid N\alpha_N \rangle \mid \alpha_i, i = 1, \ldots, N \} \]

\[ \mid i\alpha \rangle = B_{i\alpha}^+ \mid \text{vac} \rangle \quad \text{creator of excitation} \ \alpha \]

\[ H = \sum_i (E_i \alpha - E_i 0)B_{i\alpha}^\dagger B_{i\alpha} - \sum_{\langle ij \rangle, \alpha, \alpha', \beta, \beta' \neq 0} \hat{T}_{ij \alpha\alpha', \beta\beta'} B_{i\alpha}^\dagger B_{i\alpha'} B_{j\beta}^\dagger B_{j\beta'} \]

For \( U >> t, W \) (bandwidth): excitations gapped
\[ \rightarrow \text{interband transitions suppressed} \rightarrow \text{non-interacting hopping} \]

\[ H = \sum_i (E_i \alpha - E_i 0)B_{i\alpha}^\dagger B_{i\alpha} - \sum_{\langle ij \rangle, \alpha, \beta \neq 0} T_{ij \alpha\beta} B_{i\alpha}^\dagger B_{j\beta} \]

\[ T_{ij \alpha\beta} = t \psi_{i\alpha 0}^* \psi_{j 0\beta} \]

\[ \psi_{i\alpha 0} = \langle i \alpha \mid b_i \mid i 0 \rangle \]

\[ \psi_{i\alpha 0}^* = \langle i 0 \mid b_i^\dagger \mid i \alpha \rangle \]
Transport theory of Bose glass and superfluidity

Local many-body bands:

\[ \Delta n_\alpha = n_\alpha - n_0 \]

\[ \Delta n_\alpha = 1 \]

\[ n_0 = 1 \]
\[ n_0 = 2 \]

adding disorder: \( W/U = 0.6 \)

The spectra result from a combination of
- local charging energy:
- discreteness of particle number \( n_\alpha \)

\[ E_\alpha = (\epsilon_i - \mu)n_\alpha + \frac{U}{2}(n_\alpha - 1)n_\alpha \]
Superfluid transport

The superfluid current is carried by all hole-like (many-body) excitations:

\[ J_{ij} = 2 \frac{t}{\hbar} \text{Re} \int \frac{d\omega}{2\pi} b(\omega) \left[ G_{ij}^{A}(\omega) - G_{ij}^{R}(\omega) \right] \]

\[ b(\omega) = \theta(\omega) - 1 \]

The sf current can be driven by applying a phase difference between sites i and j:

\[ J_{ij} = 2 \frac{t}{\hbar} \sin(\phi_j - \phi_i) \text{Im} \int \frac{d\omega}{2\pi} b(\omega) \left[ G_{ij}^{(n)A}(\omega) - G_{ij}^{(n)R}(\omega) \right] \]

The problem of AL of BECS amounts to the AL of all hole-like many-body excitations!

\[ \rightarrow \text{In 1-band approx:} \]

**Self-consistent AL theory (VW) of many-body states**

\[ G_{jj}^{R}(\omega) = -i \langle b_j b_{j+1} \rangle = \sum_{\alpha} \left[ \frac{\psi_{j0 \alpha} \psi_{j+10 \alpha}}{\omega - (E_{j0 \alpha} - E_{j+10 \alpha}) + i\eta} - \frac{\psi_{j0 \alpha}^{*} \psi_{j+10 \alpha}}{\omega + (E_{j0 \alpha} - E_{j+10 \alpha}) + i\eta} \right] \]

\[ F_{jj}^{R}(\omega) = -i \langle b_j b_{j-1} \rangle = \sum_{\alpha} \left[ \frac{\psi_{j0 \alpha} \psi_{j-10 \alpha}}{\omega - (E_{j0 \alpha} - E_{j-10 \alpha}) + i\eta} - \frac{\psi_{j0 \alpha}^{*} \psi_{j-10 \alpha}}{\omega + (E_{j0 \alpha} - E_{j-10 \alpha}) + i\eta} \right] \]
Transport theory for Bose glass and superfluidity

Spectral densities and diffusion coefficients of hopping many-body excitations

CPA DOS

$t/U=0.015$

$t/U=0.0213$

$t/U=0.027$
Transport theory of Bose glass and superfluidity

Phase diagram

![Phase diagram graph](image-url)
The theorem of inclusions is respected.

The phase boundary reflects the charging spectrum of the on-site many-body excitations.
Disordered Bose-Hubbard model:
Local repulsion U treated exactly.

In the limit of strong on-site interaction $U \gg t, W$, the problem of Anderson localization of a BEC can be mapped onto a problem of Anderson localization of non-interacting (among each other), hopping many-body excitations, of all possible energies.

→ Selfconsistent theory of Anderson localization applicable.

For medium strong interaction $U \sim t, W$, non-local boson interaction and overlap of many-body bands become important.

→ Dephasing

Experiments: Bose glass-superfluid transition: spatial expansion experiments
Mott phase-Bose glass transition: measurement of k-space distr.