Controlling light in scattering media

« focusing light...
... and beyond »

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Acknowledgements

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Focusing coherent light through opaque strongly scattering media

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A new paradigm for light control in complex media!
What has been done already?

• Focusing
• Enhancement of fluorescence
• Opaque Lens
• Open channels

... 

...

All based on optimization method

What else can you do with a SLM and a CCD to study of control light in random medium?
Our system of study: A thick (opaque) layer of ZnO on a slab

- Very stable (stationary diffuser, stable speckle figure)
- Completely disordered, in the multiple scattering regime
- Linear, open system

A natural formalism: the scattering matrix

Link the input to the output fields of the system

\[
\begin{pmatrix}
A_{in} \\
B_{in}
\end{pmatrix}
= \begin{pmatrix}
r & t' \\
 t & r'
\end{pmatrix}
\begin{pmatrix}
A_{out} \\
B_{out}
\end{pmatrix}
\]

Scattering matrix \( S \)

The transmission matrix (TM) can be measured with a CCD and a SLM!
Overview

• Experimental measurement of the Transmission Matrix
  (we don’t have transducers!)

• Imaging applications

• Fundamental insight on the medium
N=256 macropixels/modes

Iterative focusing in a time $\propto N$ (Allard Mosk Group)

We have been able to measure the transmission matrix $K$ in $4N$ steps

$$E_{\text{out}}^n = \sum_n k_{mn} E_{\text{in}}^n$$
**Speckle interferometry**

\[ I_{out} = |E_{out}|^2 \]

Intensity measurement

Not good enough

\[ I^\phi = |E_{out} + e^{i\phi} E_{ref}|^2 \]

4 Steps to extract the amplitude of the speckle on the CCD

\[ E_{out} \propto (I^0 - I^{\pi}) + i \left( I^{3\pi \over 2} - I^{\pi \over 2} \right) \]

…but requires interferometric stability during several minutes

**Solution**

use part of the speckle as a reference (the scattering medium is the interferometer)

- Excellent stability
- The reference is a speckle field
A good basis to start

The canonical (« pixels ») basis

- Need to turn « off » pixels
- Bad SNR

The Hadamard basis

- All pixels are « on »
- Phase shift of 0 or $\pi$
- Maximal intensity and SNR

The Hadamard basis is well adapted for TM reconstruction with a phase-only SLM
Measuring the transmission matrix $K$

1) Display vector $U_k$ on the controlled part

2) Shift the global phase of the reference and take 4 speckle image

3) Retrieve the amplitude and phase of the speckle due to the $U_k$ (including reference) $E_{out},E_{ref}$

Repeat N times

$K_{obs} = K \times S_{ref}$
How does $K_{obs}$ look like?

$K_{obs} = K \times S_{ref}$

- 16x16 pixels on the SLM = 256 input modes
- 16x16 pixels on the CCD = 256 output modes
- « raster » effect due to the amplitude of $S_{ref}$
Exploiting $K_{obs}$: focusing

Can $K$ tell us what input will give a given output?

**YES:**

$$E^{in} = K^{\dagger}E^{target}$$

Two interpretations: phase-conjugation, time-reversal

With a phase-only SLM, we have to send

$$E^{in} = \frac{K^{\dagger}_{obs}E^{target}}{|K^{\dagger}_{obs}E^{target}|}$$

Initial speckle | One point focusing | Multiple point focusing

Ability to focus on any, or many points **in one step**.
Similar SNR expected as for iterative focusing (here SNR=54 for N=256 pixels)
How good is focusing? (and how physical is the measured matrix?)

Ideal phase conjugation:

\[ O_{foc}^c = KK^\dagger \]

In our case:

\[ O_{norm}^{foc} = K_{obs}K_{norm}^\dagger \]

With

\[ k_{ij}^{norm} = k_{ij}/|k_{ij}| \]

N=256 modes (=16x16 pixels on the CCD)

N=256

Expected image
Comparing experimental to expected focus

Experimental SNR = 40% of expected SNR

Optimal when pixel size matches speckle size
Imaging through a random medium: Principle

Can we determine the input image by analysing the output speckle?

=YES: in fact it is the reciprocal problem of focusing!

\[ E_{\text{img}} \approx K_{\text{obs}}^{\dagger} E_{\text{out}} = O_{\text{foc}}^{\dagger} E_{\text{obj}} \]
An amplitude object = substraction of two phase object on the SLM:

\[ \begin{array}{c}
\text{1 and -1} \\
\text{0 and 1}
\end{array} \]

**METHOD:**

1) Generate the phase object
2) Measure the output speckle
3) Apply $K^+$ and recover the reconstructed image
4) Repeat for the second phase object
5) Subtract the results to recover the intensity image
Imaging through a random medium: Result

Result for one and two pixels

Same expected SNR as for focusing

Can we reconstruct a more complex image? (we don’t know yet)
Beyond imaging: what information can we get from the TM?

A lot! for instance:

• Single and multiple scattering component, Backscattering cone (in reflexion)

• Field-field correlations

• Diffusive properties, Presence of closed and open channels, Localization?
The singular values

K is a N×N complex matrix, not hermitian

A singular value decomposition (SVD) consists in writing $K = U \Lambda V$, where $U$ and $V$ are unitary and $\Lambda$ is a diagonal matrix whose nonzero elements $\lambda_i$ are called the singular values of $K$.

The distribution of singular values $\rho(\lambda)$ is a relevant observable of transport:

- The $\lambda_i$ corresponds to the transmission values
- The corresponding $U_i$ and $V_i$ corresponds to the input and output eigenvectors
- The $\sum \lambda_i^2$ corresponds to the total transmittance for a plane wave

For a multiple scattering medium, $K$ is expected to show some universal features predicted by Random matrix theory (RMT).

A. Aubry et al., Physical Review Letters 102, 84301 (2009)

Acoustics: a multiple scattering medium has been show to follow some RMT predictions. In particular $\rho(\lambda)$ follow the so-called « quarter circle law ».
Extracting the singular values of $K$

\[ K_{obs} = K \times S_{ref} \]

Because of $S_{ref}$:
Not random (correlations)

\[ K_{fil} = K \times \frac{S_{ref}}{S_{abs}} \propto K \times \Sigma \phi \]

Compensating the amplitude of $S_{ref}$,
we recover $K$, with an unknown local phase $\Sigma \phi$

\[ \text{SVD}(K_{fil}) = \text{SVD}(K) \]
The experimental distribution of SV is in good accordance with random matrix theory.
Conclusions

Measurement of the transmission matrix of a multiple scattering medium.

0) not so difficult nor slow with current technology

1) An operative tool to manipulate light, A valid alternative to iterative techniques
   • Application to focusing
   • Application to detection

2) An important tool to study complex material experimentally
   • Demonstration of a universal behavior or RMT: the quarter circle law
   • Fill a gap between theory and experiments

\textit{Arxiv:0910.5436}

\textbf{Perspectives:}
More applications!
More fundamental measurements!
Any suggestion welcome!