Introduction

- **Strong global correlations and synchronization present**
- To introduce new quantitative measures to describe coupled-map lattices, in order to reveal the correlations;
- To investigate formation of transient patterns which are more interesting than stationary ones.

The model

A classical field $\psi(x,y,t)$ defined on a two-dimensional spatial lattice. Its evolution in (dimensionless, discrete) time $t$ is given by:

$$
\psi(x,y,t+1) = (1 - 4d)\psi(x,y,t) + d[f(\psi(x+1,y,t)) + f(\psi(x-1,y,t))] + f(\psi(x,y+1,t)) + f(\psi(x,y-1,t))
$$

where the function $f(\psi) = c\psi(1 - \psi)$, and the parameters $c$ and $d$ are constant. The set of values taken by $\psi$ is the interval $[0,1]$.

Features

- Individual (separated) map exhibit chaotic behavior for $c > 3.5699...$
- With the help of logistic maps the Feigenbaum constant has been discovered.
- Spatio-temporal chaos, and wave-like behavior well known in 1D CMLs.
- No obvious symmetries or conservation laws.

Detour: basic characteristics of condensates.

- The presence of off-diagonal long-range order (ODLRO).
- The presence of one eigenvalue of the one-particle reduced density matrix which is much larger than all other eigenvalues.
- The population of the zero-momentum mode is much larger than population of all other modes (valid for idealized systems with periodic boundary conditions).

The above characteristics acquire quantitative meaning via a density matrix and Fourier transformation of $\psi$.

Density matrix and BEC properties

Define

$$
\bar{\rho}(x',x) = \langle \sum_{y=0}^{N-1} \psi(x,y)\psi(x',y) \rangle_t,
$$

and

$$
\rho(x,x') = \bar{\rho}(x',x)/\sum_{x} \bar{\rho}(x,x).
$$

We call the quantity $\rho(x,x')$ the reduced density matrix of CML; $\rho$ is a real symmetric, positive-definite matrix with the trace equal to 1. $(\ldots)_t$ denote the time averaging:

$$
(\ldots)_t = \frac{1}{T_T} \sum_{t=T_T+1}^{T} (\ldots),
$$

$T$ is the total simulation time and $T_T$ is the averaging time. ODLRO is present in the system if $\rho(x_1 + x, x_1 - x)$ does not approach zero with increasing $x$.

We say that CML is in a "condensed state" if the largest eigenvalue of $\rho$ is significantly larger that all other eigenvalues.

Results: largest eigenvalues

We have performed our numerical experiment with six values of the non-linear parameter $c$ $(3.5 + 0.1 \cdot i, i = 0,1,...,5)$, five values of the diffusion constant $d$ $(0.05 \cdot j, j = 1,2,..,5)$, and two different initial conditions.

- Type (A) initial conditions: $\psi$ is initially "excited" only at a single point at $t = 0$: $\psi(N/2,N/2,0) = 0.5$.
- Type (B) initial conditions: at $t = 0$ $\psi$ is chosen randomly at all points.

<table>
<thead>
<tr>
<th>$d$</th>
<th>$c = 3.5$</th>
<th>$c = 3.6$</th>
<th>$c = 3.7$</th>
<th>$c = 3.8$</th>
<th>$c = 3.9$</th>
<th>$c = 4.0$</th>
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<td>0.905</td>
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<td>0.493</td>
<td>0.483</td>
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</tr>
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</table>

Results: ODLRO

- The dependence of $|\psi|$ on the discrete vector of momentum $(m,n)$ for $d = 0.20$ and $c = 3.7$; solid line: type (A) initial conditions, dashed line: type (B) boundary conditions.

Results: Dominant modes

- The values of $|\psi|$ has been normalized in such a way that $|\psi(0,0)| = 1$.

Results: Patterns

- Grayscale shaded contour graphics representing the values of the field $\psi$ after 3000 time steps for various parameters and type (A) initial conditions. Darker regions are those with higher values of $\psi$.