Transient pattern formation and condensate-like behavior in coupled-map lattices

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Introduction

- *Strong global correlations and synchronization present*
  - suggest there should be some connection with Bose-Einstein condensation
- We propose:
  - to introduce new quantitative measures to describe coupled-map lattices, in order to reveal the correlations;
  - to investigate formation of *transient* patterns which are more interesting than stationary ones.

The model

A classical field $\psi(x, y, t)$ defined on a two-dimensional spatial lattice. Its evolution in (dimensionless, discrete) time t is given by:

$$\psi(x,y,t+1) = (1-4d)f(\psi(x,y,t)) + d\,[f(\psi(x+1,y,t)) + f(\psi(x-1,y,t)) + f(\psi(x,y+1,t)) + f(\psi(x,y-1,t))] \quad (1)$$

where the function f is given by:

$$f(\psi) = c\psi(1-\psi), \quad (2)$$

and the parameters c and d are constant. The set of values taken by ψ is the interval $[0, 1]$.

Features

- Individual (separated) map exhibit chaotic behavior for $c > 3.5699...$.
- With the help of logistic maps the Feigenbaum constant has been discovered.
- Spatio-temporal chaos, and wave-like behavior well known in 1D CMLs.
- No obvious symmetries or conservation laws.

Detour: basic characteristics of condensates.

- The presence of off-diagonal long-range order (ODLRO).
- The presence of one eigenvalue of the one-particle reduced density matrix which is much larger than all other eigenvalues.
- The population of the zero-momentum mode is much larger than population of all other modes (valid for idealized systems with periodic boundary conditions).

The above characteristics acquire quantitative meaning via a density matrix and Fourier transformation of ψ.

Density matrix and BEC properties

Define

$$\bar{\rho}(x,x') = \langle \sum_{y=0}^{N-1} \psi(x,y)\psi(x',y)\rangle_t,$$

and

$$\rho(x,x') = \bar{\rho}(x,x') / \sum_x \bar{\rho}(x,x).$$

We call the quantity $\rho(x,x')$ the reduced density matrix of CML; ρ is a real symmetric, positive-definite matrix with the trace equal to 1. $\langle ...\rangle_t$ denote the time averaging:

$$\langle(...)\rangle_t = \frac{1}{T_s}\sum_{t=T-T_s}^{T}(...),$$

T is the total simulation time and T_s is the averaging time.
ODLRO is present in the system if $\rho(x_1+x, x_1-x)$. does not approach zero with increasing x.
We say that CML is in a "condensed state" if the largest eigenvalue of ρ is significantly larger that all other eigenvalues.

Results: largest eigenvalues

We have performed our numerical experiment with six values of the non-linear parameter c $(3.5 + 0.1\cdot i,\ i = 0, 1, ..., 5)$, five values of the diffusion constant d $(0.05\cdot j,\ j = 1, 2, .., 5)$, and two different initial conditions.

- type (A) - initial conditions: ψ is initially "excited" only at a single point at $t = 0$: $\psi(N/2, N/2, 0) = 0.5$;
- type (B) initial conditions: at $t = 0$ ψ is chosen randomly at all points.

Table: Largest eigenvalue of the reduced density matrix. Type (A) initial conditions

d\c	3.5	3.6	3.7	3.8	3.9	4.0
0.05	0.920	0.909	0.905	0.908	0.905	0.902
0.10	0.929	0.911	0.905	0.914	0.904	0.902
0.15	0.948	0.917	0.929	0.945	0.907	0.904
0.20	0.999	0.996	0.986	0.912	0.908	0.905
0.25	0.499	0.496	0.493	0.483	0.454	0.453

Table: Largest eigenvalue of the reduced density matrix. Type (B) initial conditions

d\c	3.5	3.6	3.7	3.8	3.9	4.0
0.05	0.920	0.909	0.904	0.903	0.902	0.902
0.10	0.999	0.998	0.986	0.959	0.903	0.901
0.15	1.0	0.998	0.986	0.967	0.905	0.903
0.20	1.0	0.998	0.986	0.967	0.905	0.904
0.25	1.0	0.998	0.986	0.966	0.905	0.904

Results: ODLRO

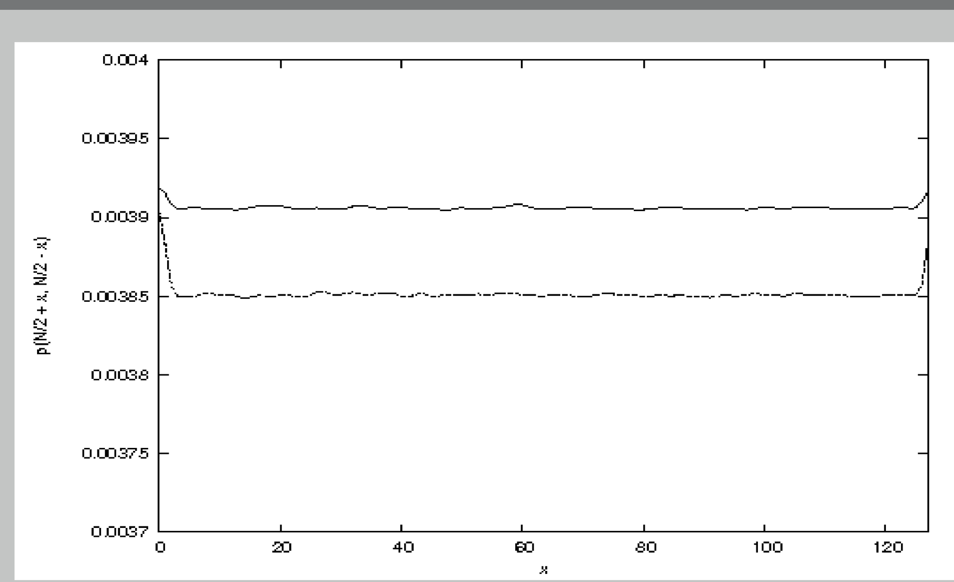

Figure: Spatial dependence of the one-particle correlation functions for $d = 0.20$ and $c = 3.7$; solid line: type (A) initial conditions, dashed line: type (B) boundary conditions.

Results: Dominant modes

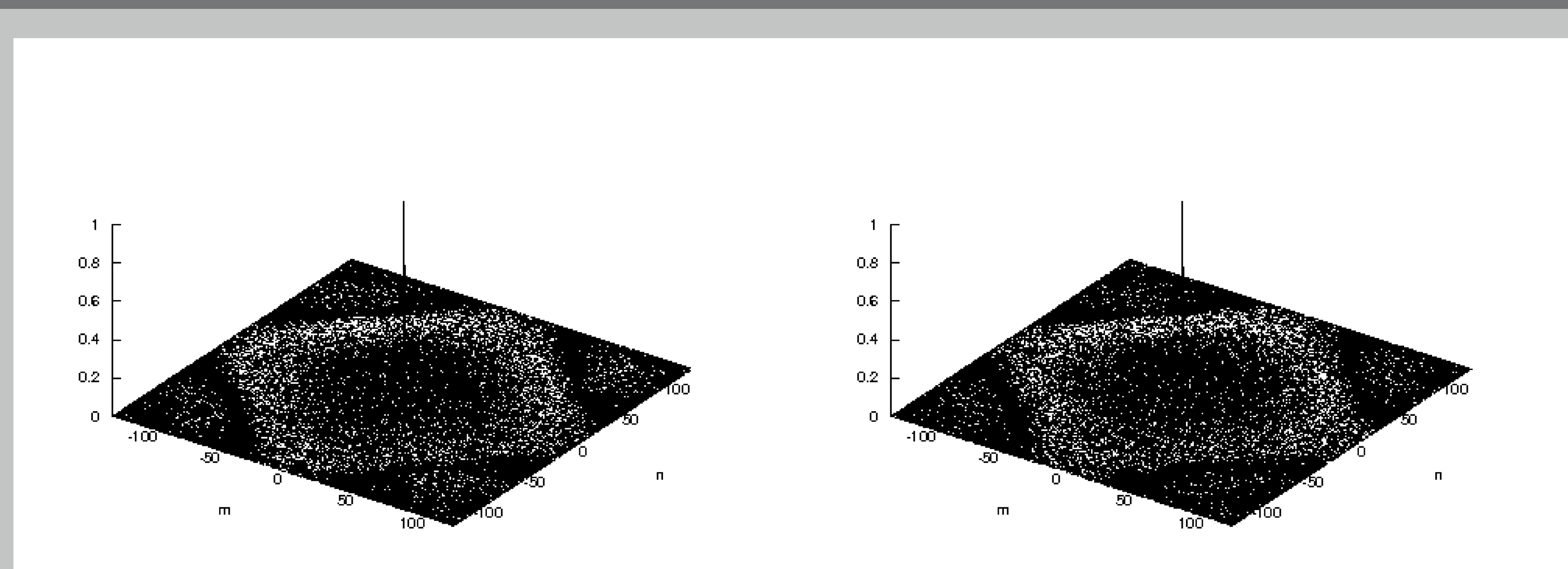

Figure: The dependence of $|\tilde{\psi}|$ on the discrete vector of momentum (m, n) for $d = 0.20$, $c = 3.7$. The values of $|\tilde{\psi}|$ has been normalized in such a way that $|\tilde{\psi}(0,0)| = 1$.

Results: Patterns

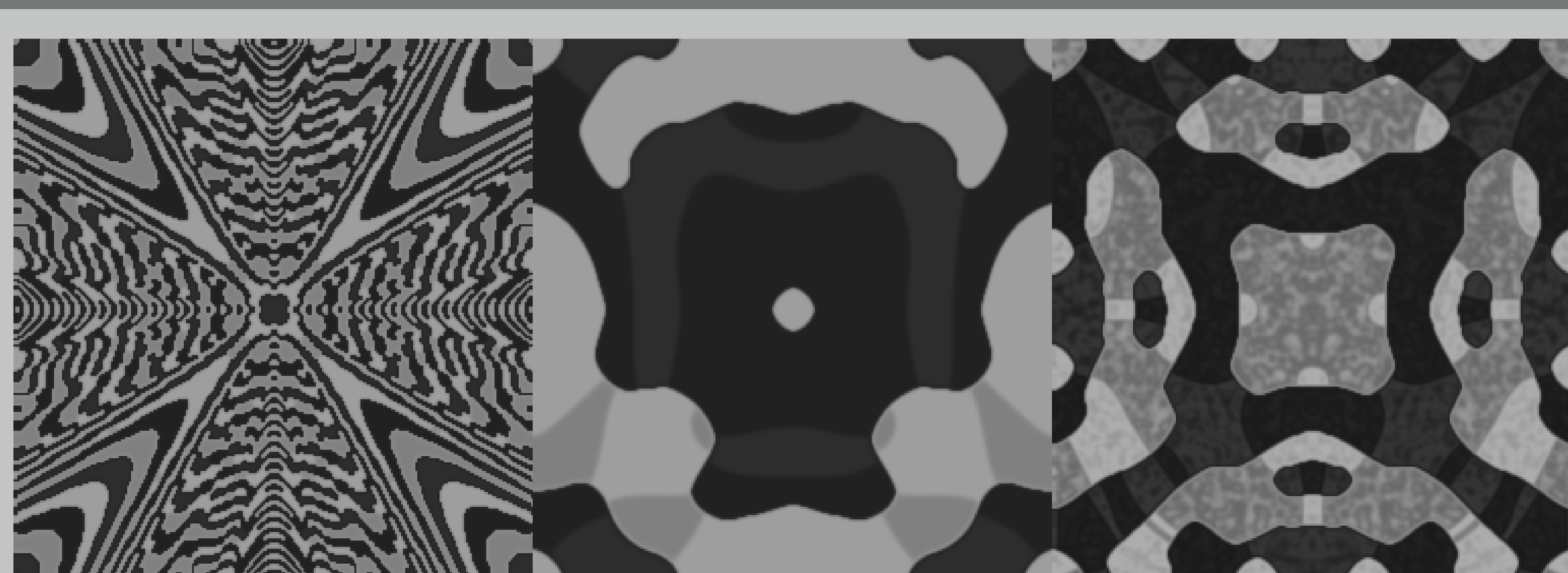

Figure: Grayscale shaded contour graphics representing the values of the field ψ after 3000 time steps for various parameters and type (A) initial conditions. Darker regions are those with higher values of ψ.