Sonic black holes and Hawking radiation in Bose-Einstein condensates

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work in collaboration with:

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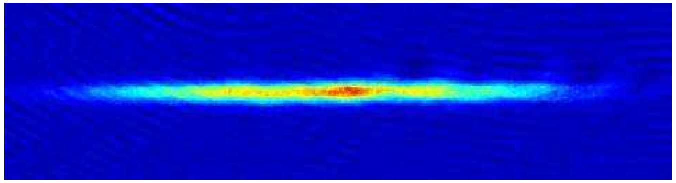
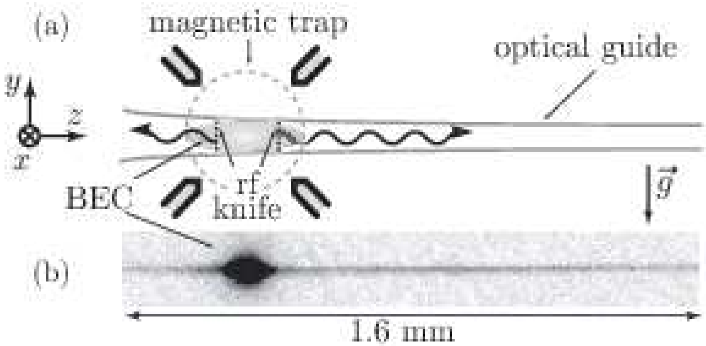
quasi-1D condensates :



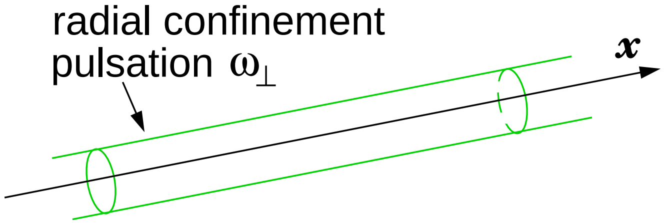
quasi-1D condensate

longitudinal size $\sim 10^2 \mu$m

transverse size $\sim 1 \mu$m



W. Guérin *et al.*, Phys. Rev. Lett. **97**, 200402 (2006)



harmonic radial confinement :

$$V_\perp(\vec{r}_\perp) = \frac{1}{2} m \omega_\perp^2 r_\perp^2 \ .$$

Mesoscopic physics & BECs : interaction in phase coherent systems, non-linear transport.

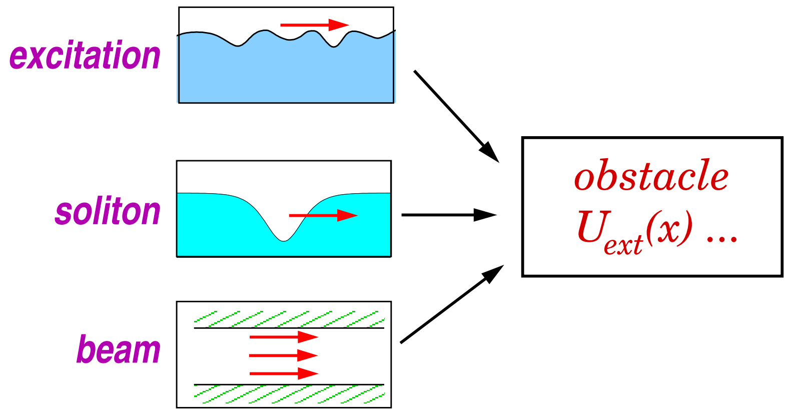
Large range of interaction regimes :

⇝ From "atom lasers" practicaly without interaction → strongly correlated 1D systems

⇝ well defined theoretical framework (Bose-Hubbard/Gross-Pitaevskii)

Situations of 1D transport :

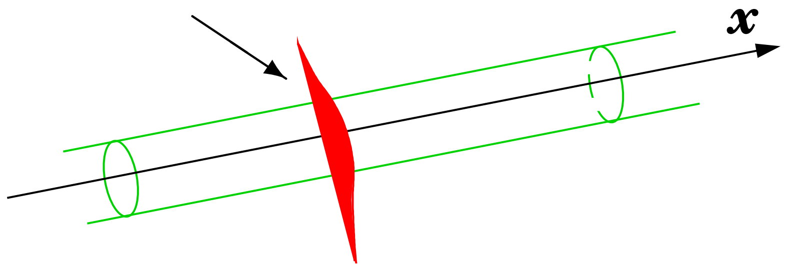
- Propagation of excitations, of (dark) solitons, of a beam ...
- In presence of localized or extended obtacles
- Effects of disorder
- Black-hole configuration
- Dispersive shock waves



1D mean field regime

Born-Oppenheimer :

$\Psi(\vec{r},t) = \psi(x,t) \times \Phi(\vec{r}_\perp;[\psi])$



1D mean field regime with order parameter $\psi(x,t)$ verifying

$$-\frac{\hbar^2}{2\,m}\partial_x^2\psi + \left(U_{\rm ext}(x) + g\,|\psi|^2\right)\psi = i\hbar\,\partial_t\psi \quad \text{or} \quad \mu\,\psi \qquad (1)$$

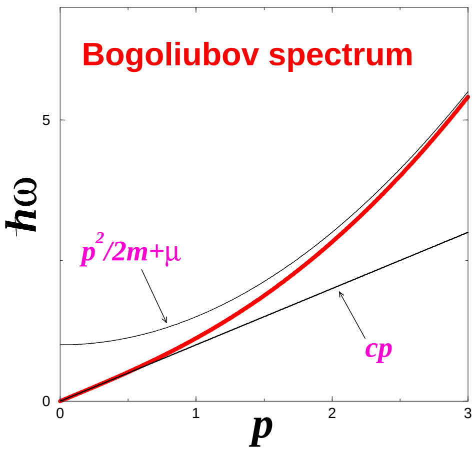
where $|\psi|^2 = n_1(x,t)$ is the longitudinal density of the condensate,
and $g = 2\,\hbar\,\omega_\perp\,a$, where a : 3D s-wave scattering length $(a > 0)$

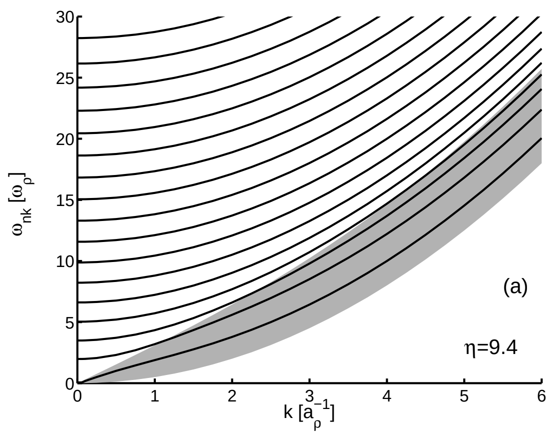
domain of validity :

$$\frac{\hbar\,\omega_\perp}{\hbar^2/ma^2} \ll n_1 a \sim \frac{\mu}{\hbar\omega_\perp} \ll 1$$

• The first inequality allows to avoid the Tonks-Girardeau regime and implies $E_{\rm int} \ll E_{\rm kin}$. Also $L_\phi \ggg \xi$ $\quad L_\phi = \xi \exp\left[\pi\sqrt{\frac{\hbar n_1}{2ma\omega_\perp}}\right]$

• the second inequality allows to avoid the 3D-like transverse Thomas-Fermi regime and implies that transverse motion is frozen



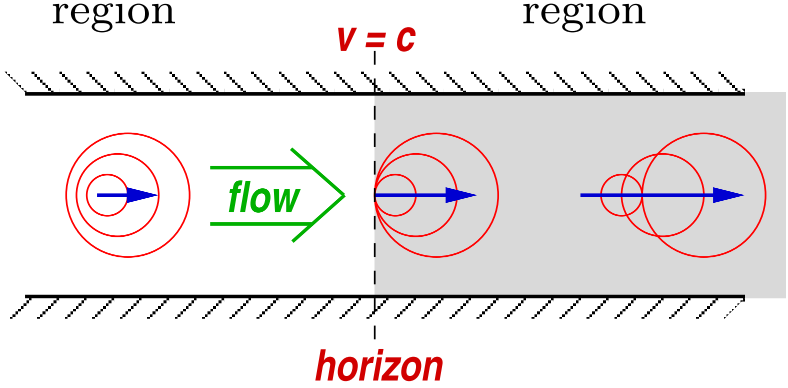


C. Tozzo & F. Dalfovo, Phys. Rev. A (2002)

$\leftarrow \eta = \mu/\hbar\omega_\perp$

only axi-symmetric excitations included $(m = 0)$

Sonic black holes : “dumb holes”



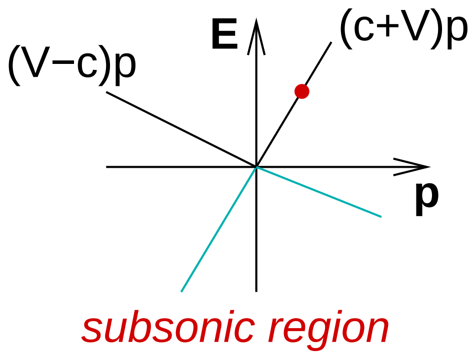
W. G. Unruh, Phys. Rev. Lett. (1981)

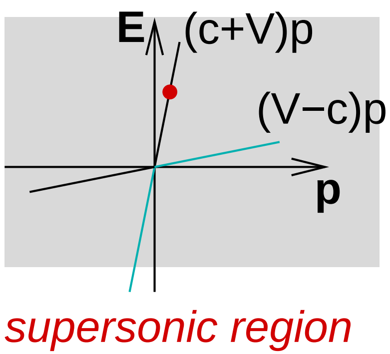
even without a source,
vacuum fluctuations $\rightsquigarrow$
Hawking radiation

in the laboratory :

$$E(p) = c\,|p| + V\,p$$

$\swarrow$ comoving $\downarrow$ Doppler

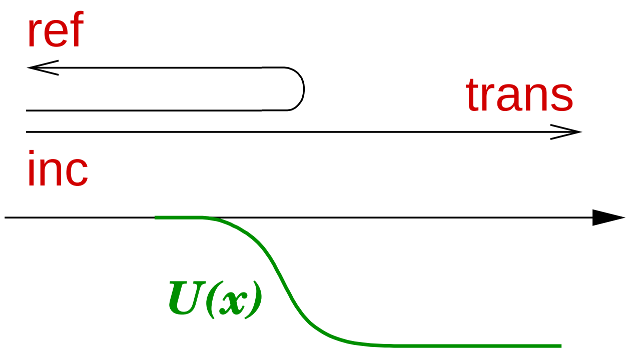




Analogous to tunnel effect : (quantum reflexion)

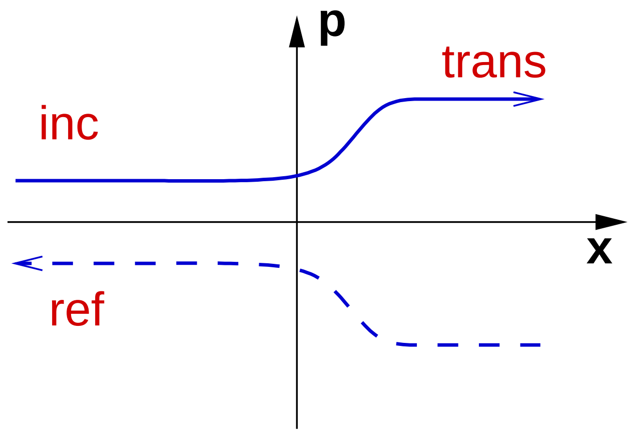
real space

particle incoming from the left with $E > U_{\max}$



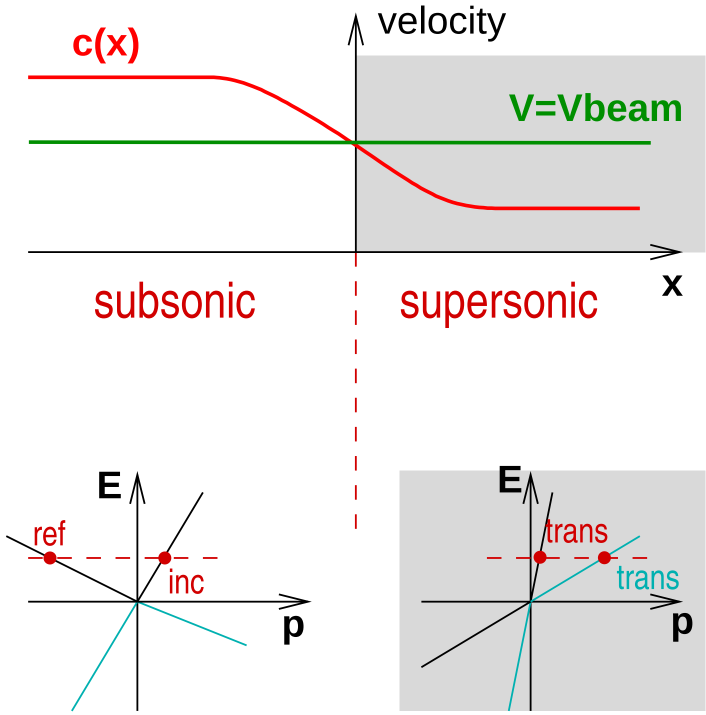
phase space trajectory

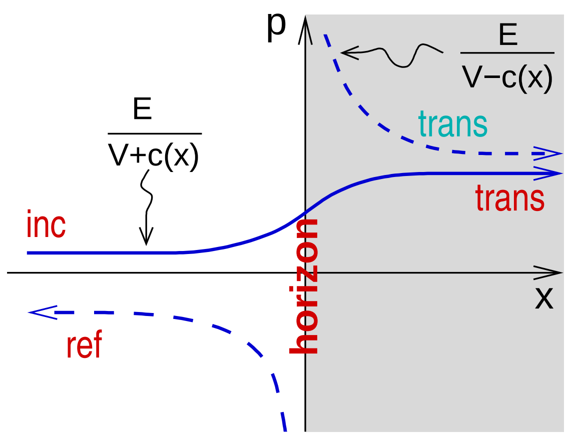
$E = p^2/2m + U(x)$



A model configuration :

$$H(x,p) = c(x)\,|p| + V\,p$$



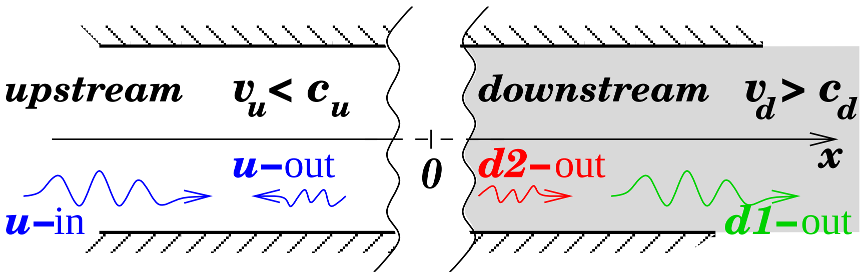


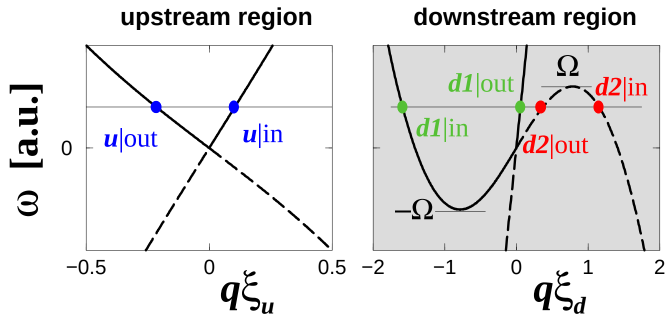
tunnel proba $R \propto \exp\left\{-\frac{2\,S}{\hbar}\right\}$

$$S = \left|\mathrm{Im}\int p(x)\mathrm{d}x\right| \simeq \frac{\pi\,E}{c'(0)}$$

of the form $R \propto \exp\left\{-E/k_B T_H\right\}$

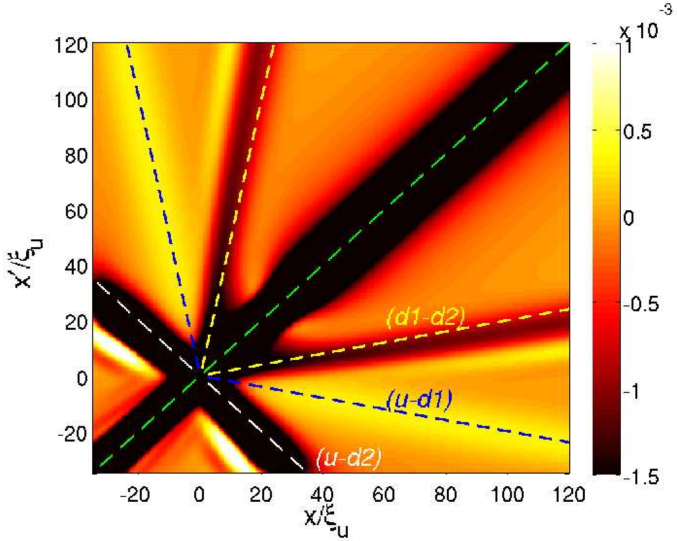
with $T_H \sim 10$ nK very weak ...





new theoretical and experimental interest :

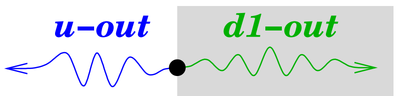
study of density correlation on each side of the horizon



$$G^{(2)}(x,x') = \frac{\langle : n(x)n(x') : \rangle}{\langle n(x')\rangle\langle n(x)\rangle} - 1$$

Balbinot, Carusotto, Fabbri, Fagnocchi, Recati
Phys. Rev. A (2008) & New J. Phys. (2008)

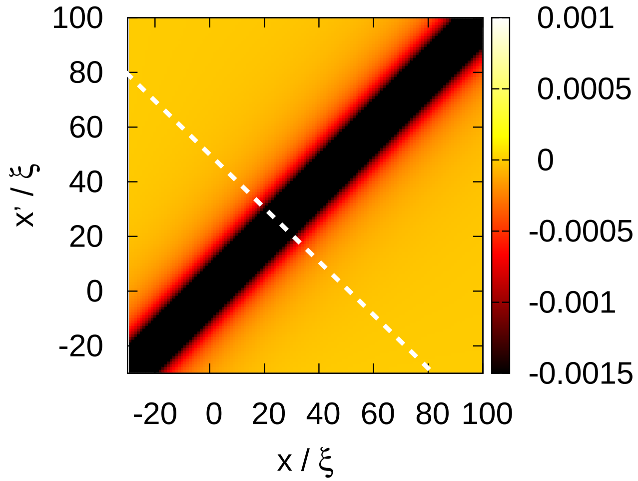
example :

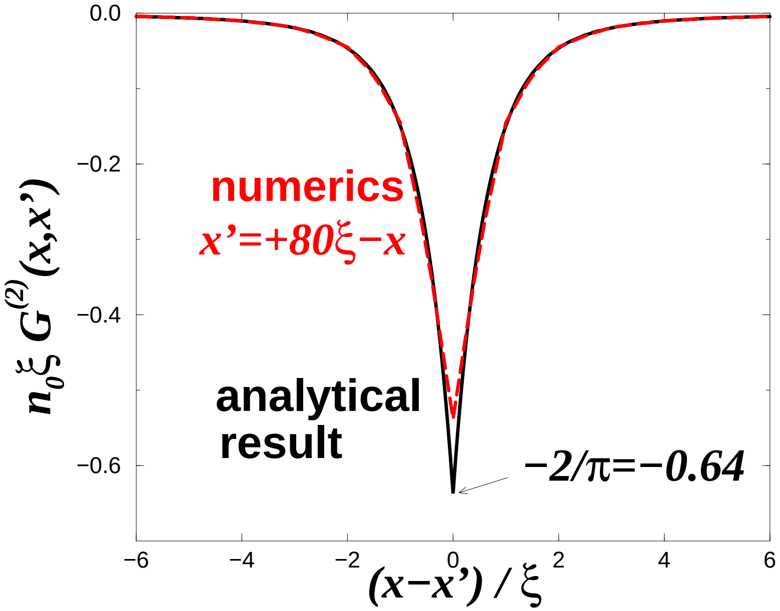


$x = (v_d + c_d)t$ correlates with $x' = (v_u - c_u)t$

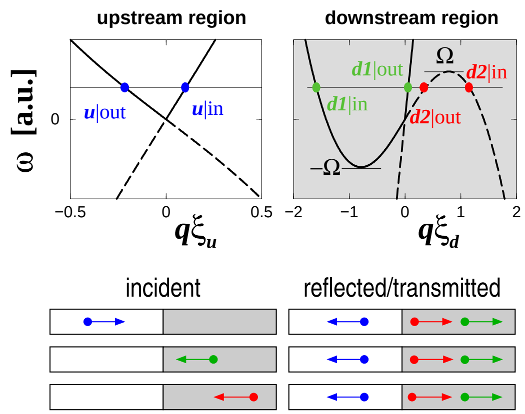
Uniform 1D Condensate (no black hole)

$$G^{(2)}(x,x') = \frac{-2}{\pi\, n_0\, \xi}\, F\left(\frac{|x-x'|}{\xi}\right) \quad \text{where} \quad F(X) = \frac{1}{2\,X}\int_0^\infty \mathrm{d}t\, \frac{\sin(2tX)}{(1+t^2)^{3/2}}\ .$$





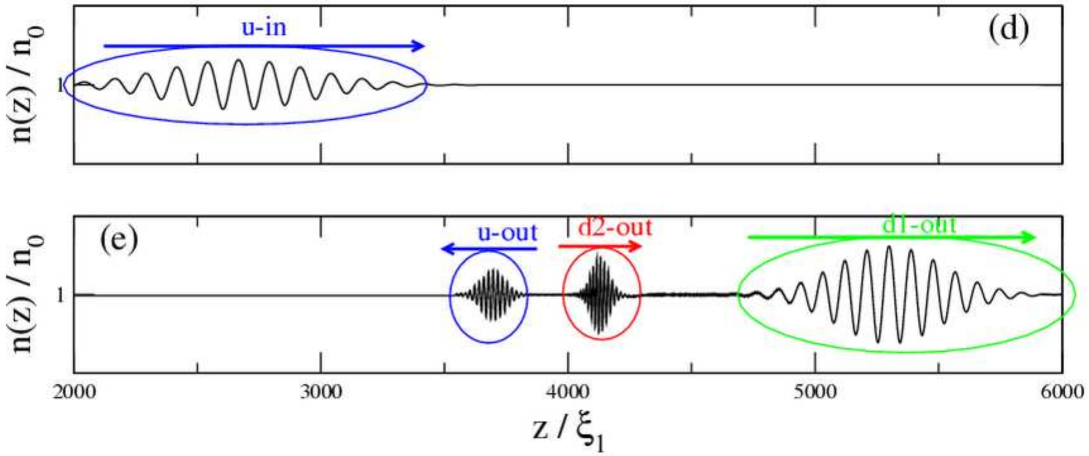
In practice :



$$(\omega - k\,V_{\rm beam})^2 = \omega_{\rm B}^2(k)$$

$$\hat\psi = \psi_0 + \delta\hat\psi \text{ with } \delta\hat\psi(x) = \sum(\,3 \text{ modes}\,)$$

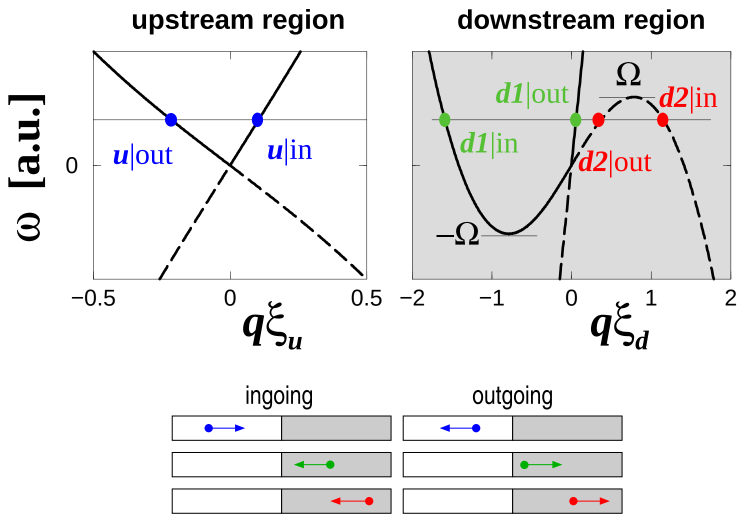
Model configuration : $U(x)$ and $g(x)$ step like with
$U(x) + g(x)n_0 = C^{\rm st}$ such that $\psi_0(x) = \sqrt{n_0}\,\exp\{ik_0x\}$, $\forall x$.



One-body Hawking signal

linear relation connecting the operators of the out-going modes $\hat{b}_{u,d1,d2}$ to the in-going $\hat{a}_{u,d1,d2}$ ones

$$\begin{pmatrix} \hat{b}_u(\omega) \\ \hat{b}_{d1}(\omega) \\ \hat{b}^\dagger_{d2}(\omega) \end{pmatrix} = \mathbf{S}(\omega) \begin{pmatrix} \hat{a}_u(\omega) \\ \hat{a}_{d1}(\omega) \\ \hat{a}^\dagger_{d2}(\omega) \end{pmatrix} .$$



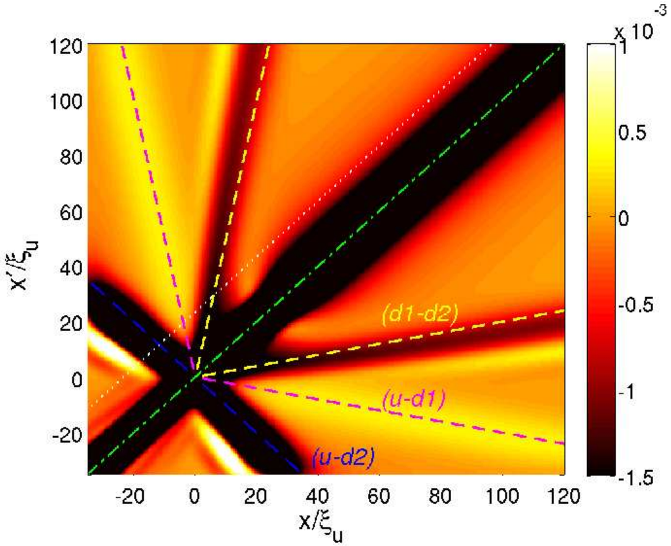
Radiation in the subsonic region occurs in the u-outgoing mode with

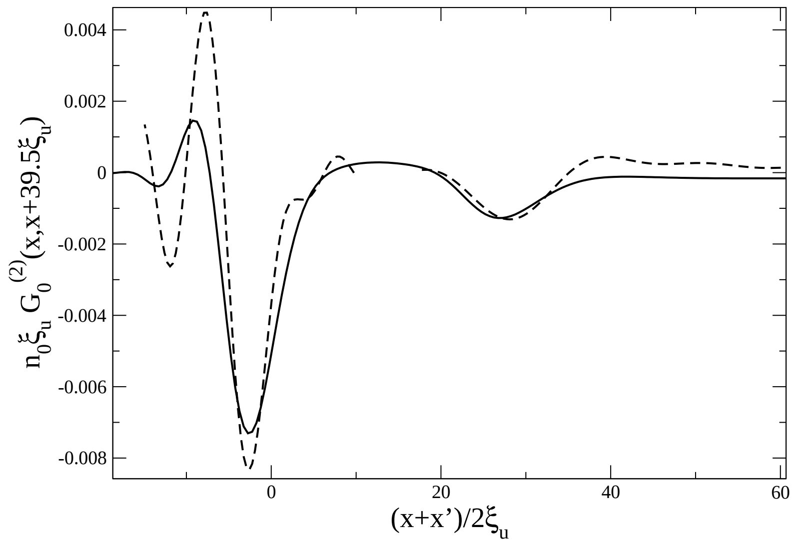
$$\frac{\mathrm{d}I_u^{\mathrm{out}}}{\mathrm{d}t\,\mathrm{d}\omega} = \langle \hat{b}^\dagger_u(\omega)\,\hat{b}_u(\omega)\rangle = |\mathbf{S}_{uu}|^2\, I_u^{\mathrm{in}} + |\mathbf{S}_{ud1}|^2\, I_{d1}^{\mathrm{in}} + |\mathbf{S}_{ud2}|^2\,(I_{d2}^{\mathrm{in}} + 1)\,.$$

at $T = 0$: $\quad \dfrac{\mathrm{d}I_u^{\mathrm{out}}}{\mathrm{d}t\,\mathrm{d}\omega} = |\mathbf{S}_{ud2}|^2$ needs $\begin{cases} u \rightleftharpoons d2 \text{ mode conversion} \\ d2\text{-ingoing mode !} \end{cases}$

Two-body Hawking signal

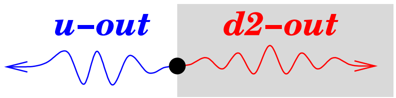
Comparison of numerical and analytic results (stationary phase neglecting interferences between the correlation signals) :





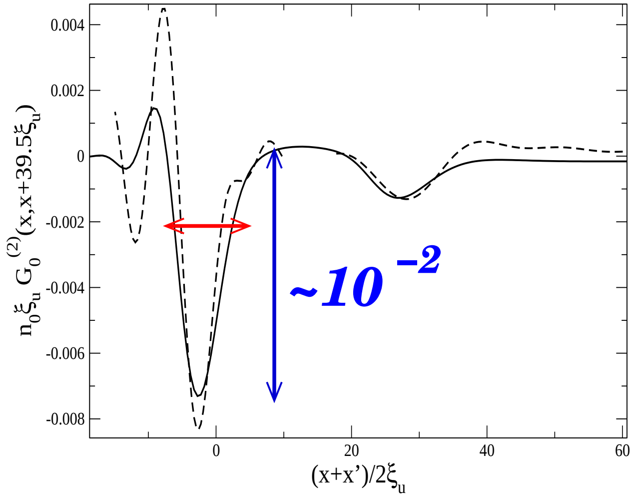
A. Recati, N. Pavloff & I. Carusotto, Phys. Rev. A **80**, 043603 (2009)

main correlation signal :



$x = V_{d2-out}\, t$ correlates with $x' = V_{u-out}\, t$

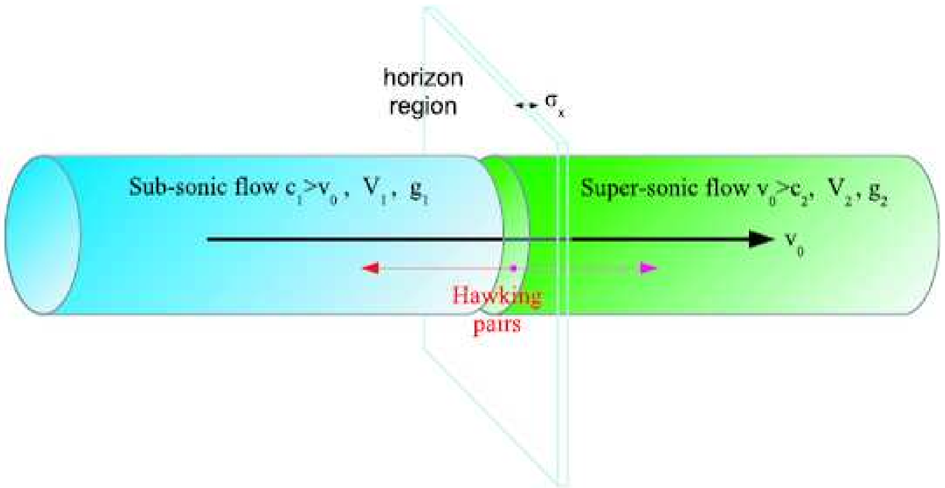
orders of magnitude :



^{87}Rb

	laser	cigar
	$n_0 \sim 50\ \mu\text{m}^{-1}$	$100\ \mu\text{m}^{-1}$
	$c \sim 1$ mm/s	3 mm/s
	$\xi \sim 1\ \mu$m	$0.1\ \mu$m

$$\left|G^{(2)}\right|_{\max} \sim 2 \times 10^{-4} \leftrightarrow 10^{-3}$$



need for new dumb-hole configurations !

Conclusion

Density correlations appear as promising tools for identifying Hawking radiation ...with some unessential limitations.

→ Clear signal , well understood. One knows where to look, and at which quantity.

→ Poorly affected by noise and finite T .

→ Drawbacks :
- weak signal intensity.
- awkward configuration.
- what about transverse degrees of freedom ?

→ What comes next ?
- more realistic dumb hole configurations,
- white hole stability ...

Conclusion (bis)

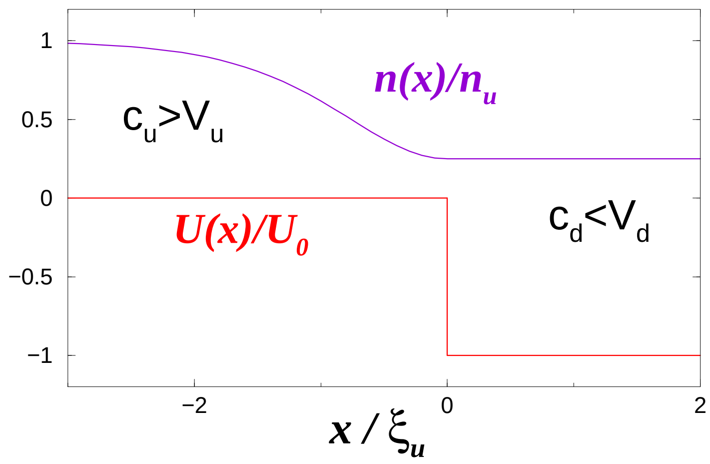
General comments concerning some of the physical problems that can be investigated by means of atom lasers.

→ transport in presence of disorder . Effects of interaction lead to qualitatively different phenomena. What is the transmission in the time dependent regime ?

→ Hawking radiation , BECs seem to offer the most promising prospect to observe a fully quantum Hawking radiation.

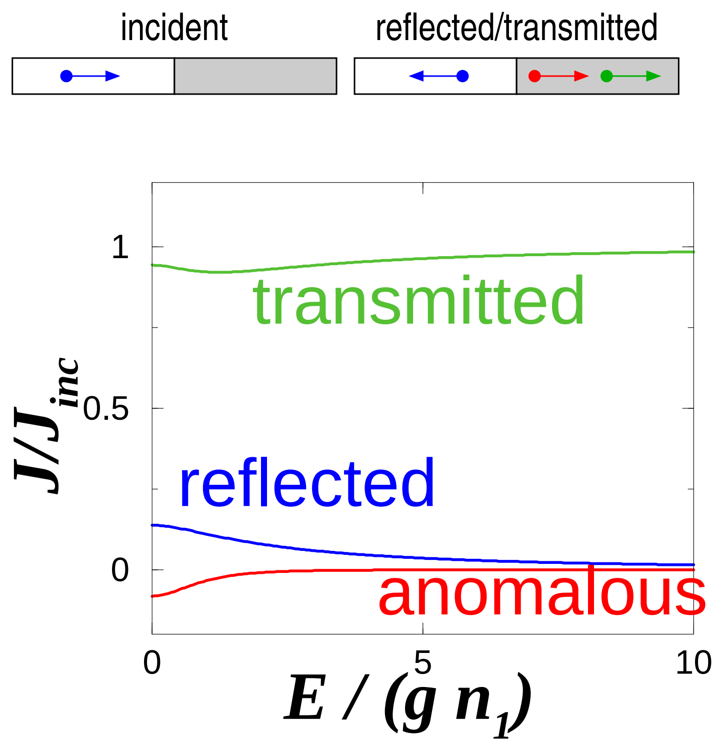
→ Dispersive shocks , BEC appears to be a versatile tool for studying frictionless nonlinear mechanisms of dissipation.

Waterfall configuration

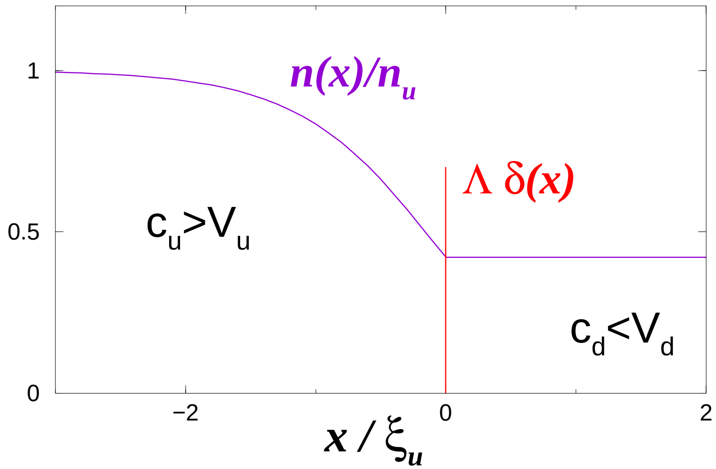


$V_d/c_d = 0.25 \qquad V_d/c_d = 16$

for instance :

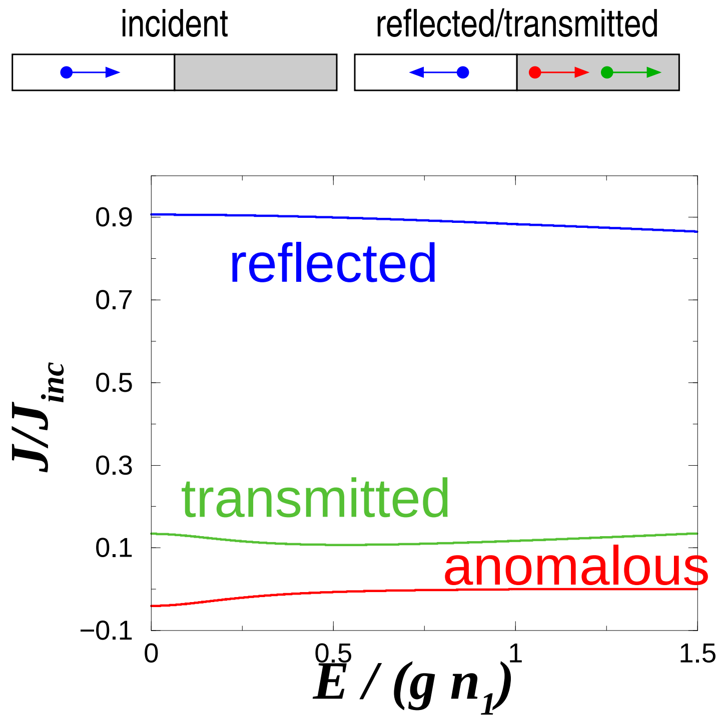


Localized obstacle



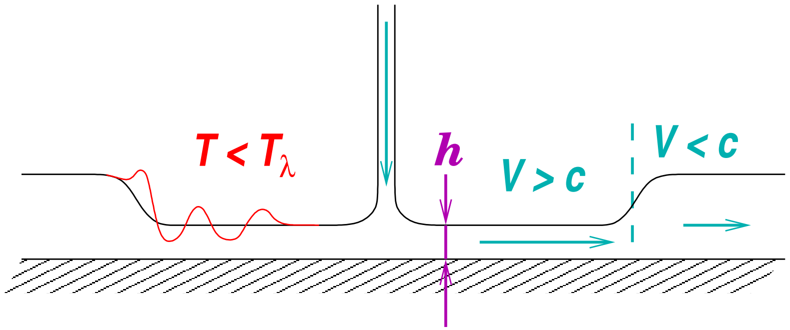
$V_u/c_u = 0.1 \qquad V_d/c_d = 5.0$

for instance :



At longer term ...

The hydraulic jump is a stable white hole (Volovik JETP 2005)



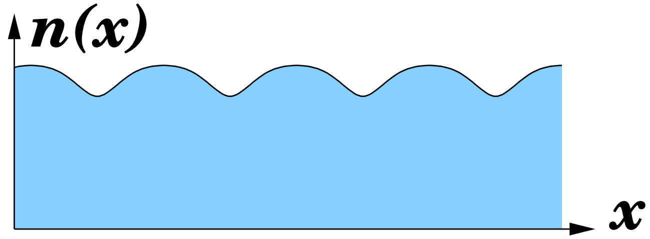
$$c = \sqrt{gh}$$

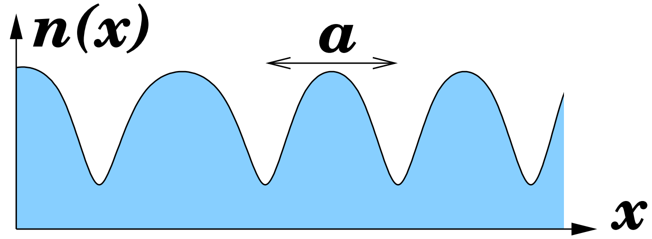
$$(\omega - \vec{k}.\vec{v})^2 = c^2\, k^2 \left[1 + h^2 k^2 \left(-\frac{1}{3} + \frac{\sigma}{\rho g h^2}\right) + ...\right]$$

appearance of oscillations in the superfluid phase ?
cf, Pitaevskii striped phase ?

1D Super-solid

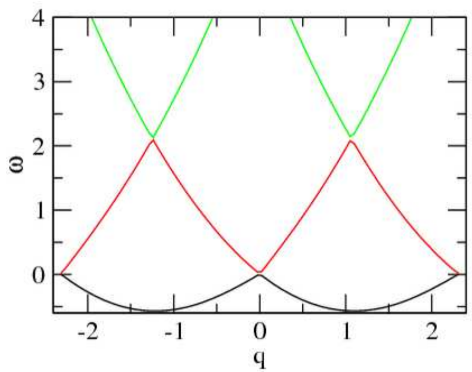
L. Pitaevskii (JETP 84): above the Landau critical velocity, a super-sonic superfluid forms a "striped phase"

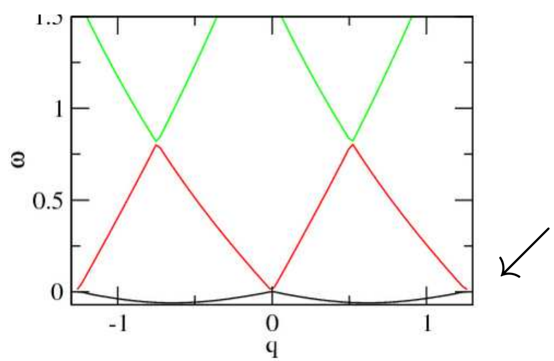




Question: is this "supersolid" phase superfluid ?

One has to study the excitation spectrum :





$$\omega \simeq -\sqrt{\frac{k}{m}}\left|\sin\left(\frac{qa}{2}\right)\right|$$